

Armed Services Technical Information Agency

AD

19377

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

DTMB/NS 802 19377

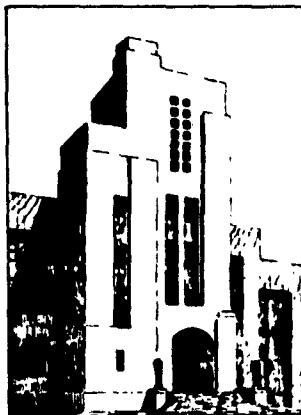
ASTIA FILE COPY

NAVY DEPARTMENT
THE DAVID W. TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

AN EXPERIMENTAL INVESTIGATION OF THE DRAG
AND SHAPE OF AIR BUBBLES
RISING IN VARIOUS LIQUIDS

by

W.L. Haberman and R.K. Morton



PROPERTY OF R.D.
TECHNICAL LIBRARY

September 1953

Report 802

NS 715-102

**AN EXPERIMENTAL INVESTIGATION OF THE DRAG AND SHAPE
OF AIR BUBBLES RISING IN VARIOUS LIQUIDS**

by

W.L. Haberman and R.K. Morton

September 1953

**Report 802
NS715-102**

TABLE OF CONTENTS

	Page
ABSTRACT	1
INTRODUCTION	1
THEORETICAL SOLUTIONS	2
DIMENSIONAL ANALYSIS	4
PREVIOUS EXPERIMENTAL WORK	
Rate of Rise of Bubbles	6
Wall Effect	7
Cylindrical Bubbles	8
SCOPE OF THE PRESENT INVESTIGATION	8
EXPERIMENTATION	
Test Tanks	10
Test Liquids	10
Bubble Generation	12
Determination of Bubble Size	12
Other Experimental Techniques	13
Motion Pictures and Their Evaluation	14
RESULTS	
Terminal Velocity of Air Bubbles	17
Wall and Proximity Effects	17
Nondimensional Presentation of Bubble Data	24
Spherical Bubbles	24
Ellipsoidal Bubbles	28
Spherical Cap Bubbles	32
Path of Bubbles	32
Bubbles in Filtered and Tap Water	37
Effect of Surface-Active Substances	39
Bubbles as Rigid Bodies	41
SUMMARY	42
ACKNOWLEDGMENTS	43
REFERENCES	43
APPENDIX - RATE OF RISE OF GAS BUBBLES IN DISTILLED WATER	47

NOTATION

A	Projected area of body
C_D	Drag coefficient $\left(-\frac{8}{3} \frac{g r_e}{U^2}\right)$
D	Drag of body
e	Coefficient of surface viscosity
g	Acceleration due to gravity
l	Length parameter of body
M	Dimensionless parameter $\left(-\frac{g \mu^4}{\rho \sigma^3}\right)$
r_e	Equivalent radius of bubble, i.e., radius of a sphere of equal volume
Re	Reynolds number $\left(-\frac{2 r_e U \rho}{\mu}\right)$
Re'	Reynolds number inside fluid body $\left(-\frac{2 r_e U \rho'}{\mu' }\right)$
U	Terminal velocity
We	Weber number $\left(-\frac{2 r_e U^2 \rho}{\sigma}\right)$
μ	Coefficient of dynamic viscosity of fluid medium
μ'	Coefficient of dynamic viscosity of fluid inside fluid body
ρ	Density of fluid medium
ρ'	Density of body
σ	Interfacial tension

ABSTRACT

In connection with other investigations at the David Taylor Model Basin, detailed information became necessary on the motion of air bubbles in variable pressure fields. Since no information on the subject was available, a fundamental study of the motion of bubbles was undertaken. As an initial step, experiments were conducted to determine the drag and shape of single air bubbles rising freely in various liquids.

The results of the experiments show that a complete description of the motion of air bubbles is not possible by use of dimensionless parameters containing the usual physical properties of the liquid (viscosity, surface tension, density). Three types of bubble shapes were observed in each liquid, namely spherical, ellipsoidal, and spherical cap. For a specific liquid, the shape of the bubble was a function of its volume.

For tiny spherical bubbles, the drag coefficients coincide with those of corresponding rigid spheres. With increase in bubble size, a decrease in the drag as compared to that of rigid spheres occurs in some liquids. Thus, the drag curves of the spherical bubbles rising in various liquids fall between two limiting curves, namely the drag curve of rigid and fluid spheres, respectively. It was not possible to determine a criterion for the transition of the bubbles from "rigid" to fluid spheres. The region of ellipsoidal bubbles extends over different ranges of Reynolds numbers for the various liquids. The drag coefficients of spherical cap bubbles are independent of bubble size and have a constant value of 2.6.

For bubbles (equivalent radius 0.03 to 0.30 cm) rising in tap water or in water containing certain surface-active substances, experiments show an increase in drag as compared to bubbles in pure water.

Results of tests to determine the effect of the container walls on the velocity of rise are presented. A description of the experimental apparatus is given. A summary of the theoretical and experimental work of other investigators is also included.

INTRODUCTION

The tests described in this report are a continuation of experiments given in a previous Taylor Model Basin report.¹ These experiments were initiated in conjunction with the work under projects NS 713-201 and NE 051-237. The present tests, continued under NS 715-102, were conducted for the purpose of investigating the motion of bubbles rising under the

¹References are listed on page 43.

influence of gravity as an initial step for obtaining information on the behavior of bubbles in variable pressure fields.

A body rising or falling under the influence of gravity reaches a constant velocity (terminal velocity) when all forces acting on it are in equilibrium:

$$\text{Drag} + \text{Buoyant force} + \text{Weight} = 0$$

For rigid bodies, the drag will, in general, be a complicated function of the geometry of the body, the velocity, and the physical properties of the medium, i.e., the density and viscosity. For fluid bodies, such as drops and gas bubbles, the function is further complicated by the fact that the body may be of changeable shape and that properties of the fluid inside the globule, such as density and viscosity, and interfacial effects may also be important factors. In general, the shape that the fluid globule assumes is some complicated function of the hydrodynamic, viscous, and interfacial forces.

The drag of fluid bodies may either be equal to (as is the case for small bubbles) or less than that of the corresponding rigid body depending upon the conditions at the interface. In the former case there exists, effectively, a rigid surface at the interface; in the latter case, the fluid particles at the boundary have, in contrast to rigid bodies, nonvanishing tangential velocities. The circulation inside the fluid body thus reduces the drag of the body.

The experiments described in this report consisted of the determination of the terminal velocity, shape, and path of single air bubbles rising freely in various liquids as a function of bubble size. The possible effect of the walls of the container on the velocity of rise of the bubble was also investigated. A summary of pertinent theoretical and experimental work of other investigators is included.

THEORETICAL SOLUTIONS

Theoretical solutions for the drag of rigid and fluid spheres, moving slowly in an infinite medium, have been obtained for the following boundary conditions at the surface of the sphere:

1. Rigid spheres

a. Stokes' solution²

(1) Velocity

(a) The velocity vanishes.

2. Fluid spheres

a. Hadamard-Rybczynski's solution^{2,3,4}

(1) Velocity

(a) The normal velocity component vanishes.

(b) The tangential velocity components at both sides of the surface are equal.

(2) Stresses

(a) The normal and tangential stresses at both sides of the surface are equal.*

b. Boussinesq's solution^{5,6}

(1) Velocity

(a) The normal velocity component vanishes.

(b) The tangential velocity components at both sides of the surface are equal.

(2) Stresses

(a) The normal stress at the inside of the surface is larger than the stress at the outside due to the dynamic surface tension.**

(b) The tangential stress at the surface is increased across the surface due to the dynamic increment of surface tension.

The drag of a sphere in an infinite medium of uniform velocity U thus becomes:

1. Rigid sphere: $D = 6\pi\mu r U$

2. Fluid sphere: a. $D = 6\pi\mu r U \frac{2\mu + 3\mu'}{3\mu + 3\mu'}$

b. $D = 6\pi\mu r U \frac{e + r(2\mu + 3\mu')}{e + 3r(\mu + \mu')}$

where D is the drag, μ is the coefficient of viscosity of the medium, r is the radius of the sphere, μ' is the coefficient of viscosity of the fluid inside the sphere, and e is the coefficient of surface viscosity.

Using the condition of equilibrium for a sphere rising under the influence of gravity, we obtain for the rigid case:

$$6\pi\mu r U = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g$$

$$U = \frac{2}{9} \frac{r^2 g}{\mu} (\rho - \rho') \quad \text{Stokes' Law}$$

*The pressure increase across the surface due to surface tension ($= \frac{2\sigma}{r}$) was neglected in Hadamard's analysis. Inclusion of this pressure drop in the boundary condition for the normal stress does not change the results. That is to say, surface tension as manifested only in a pressure increase inside the fluid sphere does not affect its motion. (This result is also obtained by putting, in Boussinesq's analysis, the coefficient of surface viscosity, see subsequent footnote, equal to zero.)

**Boussinesq assumed that a dynamic surface tension exists at interfaces in motion. Its magnitude is given by the sum of the usual (static) surface tension and the dynamic increment. The dynamic increment varies over the surface of the sphere and at a given point is proportional to the rate of dilatation at that point. The constant of proportionality is called (due to its similarity to the viscosity coefficient) the coefficient of surface viscosity. (Surface viscosity has the dimensions mass/time, while the dimensions for viscosity are mass/length \times time.)

where ρ is the density of the medium and ρ' is the density of the sphere.

For the fluid sphere:

$$U = \frac{2}{9} \frac{r^2 g}{\mu} (\rho - \rho') \frac{3\mu' + 3\mu}{3\mu' + 2\mu} \quad \text{Hadamard-Rybczynski's Law}$$

$$U = \frac{2}{9} \frac{r^2 g}{\mu} (\rho - \rho') \frac{e + 2r(\mu + \mu')}{e + r(2\mu + 3\mu')} \quad \text{Boussinesq's Law}$$

For the case of bubbles, where $\mu' \ll \mu$ and $\rho' \ll \rho$, the last two expressions reduce to:

$$U = \frac{1}{3} \frac{r^2 g \rho}{\mu}$$

and

$$U = \frac{2}{9} \frac{r^2 g \rho}{\mu} \frac{e + 3r\mu}{e + 2r\mu}$$

In the last equation, the factor $\frac{e + 3r\mu}{e + 2r\mu}$ approaches 1 for r approaching zero or e very large; it approaches 3/2 for $e \ll r\mu$, i.e., for large r or for e approaching zero. Hence, for very small bubbles, Boussinesq's solution approaches Stokes' law as a limit, while the other limit is Hadamard-Rybczynski's solution.

From the boundary conditions as stated above, it is obvious that, for the Hadamard-Rybczynski and Boussinesq solutions, circulation exists inside the bubble. For the Stokes solution, of course, there is no circulation inside the sphere.

DIMENSIONAL ANALYSIS

Since an analytical solution for the drag of fluid bodies over a large range of sizes is hardly attainable, dimensional analysis of the phenomenon may serve to correlate the experimental results.

In the case of fluid bodies the following physical variables are usually considered as pertinent:

U	Velocity of the body
g	Acceleration due to gravity
ρ	Density of the fluid medium
ρ'	Density of the fluid body
l	Length parameter of the fluid body
μ	Coefficient of dynamic viscosity of the fluid medium

- μ' Coefficient of dynamic viscosity of the fluid inside the fluid body
 σ Interfacial tension*

The complete set of dimensionless products will therefore contain five such products. In principle, it is immaterial which complete set is chosen for the representation of the phenomenon. For example, we may use

$$f_1(C_D, Re, We, \frac{\mu}{\mu'}, \frac{\rho}{\rho'}) = 0$$

or

$$f_2(C_D, Re, M, Re', \frac{\rho}{\rho'}) = 0$$

and so on, where C_D is the drag coefficient,

Re is the Reynolds number $\left(= \frac{2lU\rho}{\mu} \right)$,

We is the Weber number $\left(= \frac{2lU^2\rho}{\sigma} \right)$,

Re' is the Reynolds number inside the fluid body $\left(= \frac{2lU\rho'}{\mu'} \right)$ and

M is a dimensionless parameter $\left(= \frac{g\mu^4}{\rho\sigma^3} \right)$

If the density and viscosity of the gas inside a bubble are considered negligible, the physical variables are reduced to six. The dimensionless products then take the form of

$$f_3(C_D, Re, We) = 0$$

or

$$f_4(C_D, Re, M) = 0$$

or

$$f_5(C_D, We, M) = 0$$

etc.

When experimental data on gas bubbles are plotted in terms of dimensionless products, complete correlation will be obtained provided the variables chosen are all the variables upon which the phenomenon depends.

In the case of bubbles, it is most convenient to use a length parameter which is based on its volume rather than a physical dimension as is customary for rigid bodies. The length parameter chosen is the equivalent radius r_e where

$$r_e = \sqrt[3]{\frac{\text{volume}}{\frac{4}{3}\pi}}$$

*The coefficient of surface viscosity is not included since there is no experimental evidence that dynamic surface tension, as postulated by Boussinesq, exists.

For bubbles rising at their terminal velocity the drag coefficient C_D can then be written as

$$C_D = \frac{(8/3) g r_e}{U^2}$$

instead of its usual form of $D/(1/2)\rho U^2 A$ where A is the projected area. For the analytical solutions of Stokes and Hadamard-Rybczynski the drag coefficient becomes, respectively:

$$C_D = \frac{24}{Re} \quad (\text{Rigid sphere})$$

$$C_D = \frac{16}{Re} \quad (\text{Fluid sphere})$$

For the special case in which only four variables, namely the velocity, the acceleration of gravity, the density of the fluid medium, and the equivalent radius, are taken as pertinent, only one dimensionless group, the drag coefficient, is obtained, i.e.,

$$C_D = \text{constant}$$

This solution will be shown to apply to the region of very large (spherical cap) bubbles.

PREVIOUS EXPERIMENTAL WORK

RATE OF RISE OF BUBBLES

Interest in the motion of air bubbles has existed for many years. The work on bubbles has, however, been mostly experimental in nature. Exceptions are an attempted theoretical analysis by Theremin⁷ in 1829, the analytical solution of Hadamard,^{2,3} Rybczynski,⁴ and Boussinesq,^{5,6} and the dimensional analysis of Schmidt⁸ and Rosenberg.¹

The early experimental work on bubbles was largely concerned with very small bubbles and was carried out for the purpose of determining the extent of Stokes region. Allen⁹ determined the rate of rise of air bubbles in water and in aniline up to bubble radii of 0.04 and 0.06 cm, respectively. Arnold¹⁰ measured velocities of small air bubbles in olive oil and in aniline. Bond and Newton¹¹ investigated air bubbles in syrup and in water glass (sodium silicate).

The range of bubble sizes was extended by other investigators^{12,13} who were mainly interested in the problem in connection with air-lift pumps,¹⁴⁻¹⁷ gas absorption,¹⁸⁻²⁰ or propagation of sound in liquids.²¹ These experiments were carried out in water. In subsequent years, some investigations were also made in liquids other than water. Davies and Taylor²² used nitrobenzene as well as water and measured velocities of large bubbles. Temperley and Chambers²³ extended the range of Taylor's experiments in water to bubbles of equivalent radii up to approximately 6 cm. Bryn²⁴ made tests in various water-glycerine and water-ethyl alcohol mixtures. Robinson²⁵ measured the rate of rise of small air bubbles

in lubricating oils. His results, however, show considerable scatter. Reports by Pickert,²⁶ Pekeris,²⁷ Worster,²⁸ and Datta et al²⁹ give summaries of the results of experiments of other investigators.

Gorodetskaya³⁰ investigated the effect of surface-active substances on the rate of rise of air bubbles in water. Further tests on air bubbles are reported in References 31-36. In addition, a limited number of tests using gas bubbles of oxygen, nitrogen, and a mixture of carbon dioxide and oxygen have been carried out in artificial sea water.^{37,38} A number of tests with oxygen bubbles were also conducted in water³⁷ and in aqueous solutions of sodium hydroxide.³⁹ Recently, Stuke⁴⁰ investigated the rate of rise of oxygen bubbles in pure (presumably distilled) water and in water containing surface-active substances. The results of the tests with gas bubbles (given in the Appendix) show no significant change in the rate of rise of the bubbles with change in the gas inside the bubble.

Because of the scatter of previous results of experiments on the rate of rise of air bubbles in water, Rosenberg¹ repeated these tests for a large range of air bubble sizes. He showed the geometric similarity between large bubbles of spherical cap shape and suggested the use of three dimensionless parameters, the drag coefficient, the Reynolds number, and the parameter M for describing bubble motion in liquids.

WALL EFFECT

Previous investigations on the motion of air bubbles in liquids were, with a few exceptions, conducted in containers of limited dimensions. Only for very large bubbles did Exner¹³ and Bryn²⁴ make their measurements in lakes. Inasmuch as the effect of the walls of the container on the rate of rise of bubbles was unknown, it was generally neglected. Miyagi¹² conducted a few tests in containers of different sizes and found that a reduction of 4 percent in the rate of rise occurred for the range of bubble sizes investigated. Dubs⁴¹ derived, from energy considerations, an analytical expression for the wall effect and concluded that a bubble of the same radius as its cylindrical container has a velocity of rise of zero. It is clear that this conclusion is in error, as experiments on cylindrical bubbles have shown.

As indicated previously, no analytical solution has yet been obtained for flows beyond the region of slow flow; consequently, the much more difficult problem of also including a finite boundary in the equations of motion becomes less capable of solution. For very slow flow about rigid spheres moving in an infinite cylindrical container, Ladenburg⁴² obtained an analytical solution for the effect of the boundary on the drag and, consequently, the velocity of the sphere. McNown et al⁴³ arrived experimentally at a wall correction coefficient for rigid spheres descending in a cylindrical container. Since air bubbles of small volume rising in water behave essentially like rigid spheres, this correction factor may be applied to such bubbles as long as the flow is still in the Stokes region.

With the exception of a number of tests^{16,17} for large bubbles, no data concerning the effect of the boundary on the rate of rise of gas bubbles beyond the region of slow flow are

available.* However, Möller⁴⁴ showed, by means of a dye technique, that the flow about a rigid sphere at a lower Reynolds number and larger boundary dimensions was identical to that at a higher Reynolds number and smaller boundary dimensions and therefore that the effect of the walls was to stabilize the flow about the sphere. These results for rigid spheres beyond the Stokes region of flow at least suggested the possibility of a similar effect for the motion of gas bubbles.

CYLINDRICAL BUBBLES

At this point, it may be of interest to mention experiments on a special form of finite boundary dimensions, i.e., the case of cylindrical bubbles. This term was first used by Gibson⁴⁵ and applies to the type of bubble formed when a long cylindrical tube filled with liquid is emptied from below or when a large amount of air is introduced through the bottom of the tube.

Gibson investigated the velocity and shape of these bubbles in water. Ward and Kessler¹⁶ conducted tests in pipes of various diameters. Hattori⁴⁶ was interested in the problem in connection with the possibility of evaluating the surface tension of a liquid. Hence, he was concerned with tubes of small diameter, since the so-called critical tube diameter (below which the bubble no longer rises but remains stationary) is a function of the surface tension of the liquid. Dumitrescu⁴⁷ obtained an analytical expression for the velocity of a cylindrical bubble by neglecting viscous and surface tension forces, thus reducing the problem to one of potential flow. The differential equation for the velocity potential together with the existing boundary condition yields a solution for the velocity of rise as a function of the tube diameter only. His experimental tests in water show that for a tube of sufficiently large diameter (3 cm for water at room temperature) the measured velocities agree very closely with the theoretical values. Therefore, for large bubbles, the physical properties of the liquid no longer have any effect on the flow about the bubble and the bubbles are geometrically similar. Davies and Taylor²² investigated the shape and rate of rise of cylindrical bubbles in order to obtain a better understanding of the pressure distribution of spherical cap bubbles in an infinite medium.

SCOPE OF THE PRESENT INVESTIGATION

The present investigation was initiated in connection with a program of study of the behavior of air bubbles in water at variable pressure gradients. Since extensive experimentation was required for direct experimental study of the motion of air bubbles in such pressure

*A paper by Coppock and Meiklejohn³⁴ has recently come to the attention of the authors. From tests conducted with air bubbles in water, they conclude that no wall effect exists for bubbles ranging in equivalent radius from 0.01 to 0.1 cm rising in a tube of 5 cm diameter.

fields,* an alternate approach appeared more feasible. It consists of calculating the motion of the bubbles in variable pressure fields from a knowledge of bubble drag at various constant pressure gradients. Experimental data on the drag of air bubbles at various pressure gradients are essential in this procedure. However, only data on the motion of bubbles in pressure gradients produced by gravity were available. Therefore, information on bubble motion in water at pressure gradients other than gravity became necessary. This information could be obtained by investigating the rise of bubbles in liquids having the same physical properties as water with the exception of the density. With all other properties of the liquids identical, varying the density would be equivalent to varying the pressure gradient. This approach, however, is not practicable, since there are no liquids available which possess such properties. The other approach is to investigate the rise of air bubbles in various liquids having different physical properties and then to attempt to correlate the results in terms of nondimensional parameters. The available information on the rise of bubbles in different liquids was too meager to allow definite conclusions regarding the significance of the parameters suggested by Reference 1. The present investigation was therefore initiated with the purpose of determining the nondimensional parameters for bubble rise by investigating bubble motion in a number of liquids of different physical properties. If it were found that the motion of air bubbles rising freely in a liquid, that is to say the motion in the pressure field produced by gravity, could be described, for example, in terms of the drag coefficient, the Reynolds number, and the parameter $M(= g\mu^4/\rho\sigma^3)$,** the results thus obtained could be used in evaluating the drag of bubbles in water at pressure gradients other than that produced by gravity. To do this it would have to be shown that the nondimensional parameters used for the freely rising bubbles are also applicable to other pressure fields. This might be accomplished, for example, by comparing the results of a bubble experiment in water in a nongravity pressure gradient with the results of bubbles rising freely in various liquids at identical "M" number.

By conducting the tests on the rise of bubbles in various liquids in a large tank, the possibility of the effect of the tank walls on the velocity of the bubbles is eliminated. Since the high cost of many desirable liquids makes the use of a large tank impractical, it became necessary to determine the possible effect of the walls on the velocity over the range of bubble sizes to be tested. This investigation consequently acquired two purposes:

1. The determination of the effect of variation of liquid properties on the motion of air bubbles.
2. The evaluation of wall effect.

*Exploratory experiments of such a nature are reported in Reference 35.

**This parameter is given in a more general form as $(\mu^4 V_p)/(\rho^2 \sigma^3)$ (where V_p is the pressure gradient; for a gravity field $V_p = \rho g$). Therefore, for a specific liquid, it is proportional to the pressure gradient.

EXPERIMENTATION

The experimental study consisted of measuring the terminal velocity of individual bubbles of various sizes rising in eight liquids. It also included the determination of the effect of the walls of the container on the bubble velocity. The experimental apparatus consisted of test tanks, means for determining the physical properties of the liquids, a regulated bubble supply, and means for measuring bubble size and velocity. Details of the experimental apparatus and procedure, the generation of the bubbles, and the test liquids are given in the following paragraphs.

TEST TANKS

In order to obtain free bubble rise and to reduce the effects of such boundaries as the bottom of the tank and the free liquid surface, the containers for the liquids had to be at least 2 ft high. The tests were performed in three transparent wall tanks; the large one was of 3 × 3 ft cross section and 5 ft height, the medium one was of 1 × 1 ft cross section and 3 1/2 ft height, and the small one was of 6 × 6 in. cross section and 2 ft height. In addition, tests were also performed in an insert of 6 × 6 in. cross section and 20 in. height, placed in the center of the medium tank.

Since the large tank was of sufficiently large dimensions, no significant wall effect was expected.* The dimensions of the medium tank were chosen large enough to reduce wall effects, yet small enough to allow use of a variety of liquids. The small tank and the insert provided an additional tank size. It was intended that if wall effect existed, the results obtained in the finite containers would be extrapolated to the case of an infinite medium.

TEST LIQUIDS

The eight test liquids used were water (at three different temperatures), Varsol,** methyl alcohol, turpentine, water containing 0.42 percent (by volume) Glim,*** mineral oil, and two corn syrup-water mixtures. Turpentine was selected as one of the test liquids because at room temperature it has the same viscosity as cold water. One of the corn syrup mixtures had approximately the same viscosity as the mineral oil (see Table 1).

The viscosity of the liquids was measured by means of ordinary and modified Ostwald viscosimeters. The accuracy of measurement of viscosity was 1.5 percent and 0.5 percent,

*A few of the previous Taylor Model Basin tests¹ were repeated in the large tank to observe any change in results. These previous tests were conducted in a tank of 4 1/2 × 25 ft cross section and 9 ft height with 8 ft depth of filtered water at room temperature, using one end of the tank for the tests.

**A trade name (Standard Oil Company) for mineral spirits (heavy naphtha), a petroleum distillate.

***Glim (Antarox A-480), a surface-active agent, is the trade name (E. T. Babbitt, Inc.) of a non-ionic, liquid detergent, a condensation product of ethylene oxide and lauryl alcohol.

TABLE 1

Summary of Liquid Properties

Liquid	Temperature deg C	Viscosity μ poises	Density ρ gm/cc	Surface Tension σ dynes/cm	"M" $\frac{g\mu^4}{\rho\sigma^3}$
Water	19	0.0102	0.998	72.9	0.26×10^{-10}
Water	21	0.0098	0.998	72.6	0.24×10^{-10}
Cold Water	6	0.0147	0.999	74.8	1.08×10^{-10}
Hot Water	49	0.0056	0.989	68.1	0.307×10^{-11}
Glim Solution	19	0.0103	1.000	32.8	2.78×10^{-10}
Mineral Oil	27.5	0.580	0.866	20.7	1.45×10^{-2}
Varsol	28	0.0085	0.782	24.5	4.3×10^{-10}
Turpentine	23	0.0146	0.864	27.8	24.1×10^{-10}
Methyl Alcohol	30	0.0052	0.782	21.8	0.89×10^{-10}
62 percent Corn Syrup and Water	22	0.550	1.262	79.2	0.155×10^{-3}
68 percent Corn Syrup and Water	21	1.090	1.288	79.9	0.212×10^{-2}
56 percent Glycerine and Water (Bryn ²⁴)	18	0.0915	1.143	69.9	1.75×10^{-7}
42 percent Glycerine and Water (Bryn ²⁴)	18	0.043	1.105	71.1	4.18×10^{-8}
13 percent Ethyl Alcohol and Water (Bryn ²⁴)	22	0.0176	0.977	43.5	1.17×10^{-8}
Olive Oil (Arnold ¹⁰)	22	0.73	0.925	34.7	0.716×10^{-2}
Syrup (Bond ¹¹)	17	180	1.48	91	0.92×10^6

respectively. The surface tension was determined by the capillary-rise method (accuracy of measurement: 3 percent) and the specific gravity of the liquids was obtained by means of hydrometers (accuracy of measurement: 1 percent). These physical properties were measured following the completion of each test. They are summarized in Table 1 together with those of liquids used by several other investigators.^{10,11,24}

For Varsol and water (room temperature and hot), tests were conducted in all three tanks; for cold water and mineral oil, in the medium tank and insert; and for all other liquids, in the small tank only.

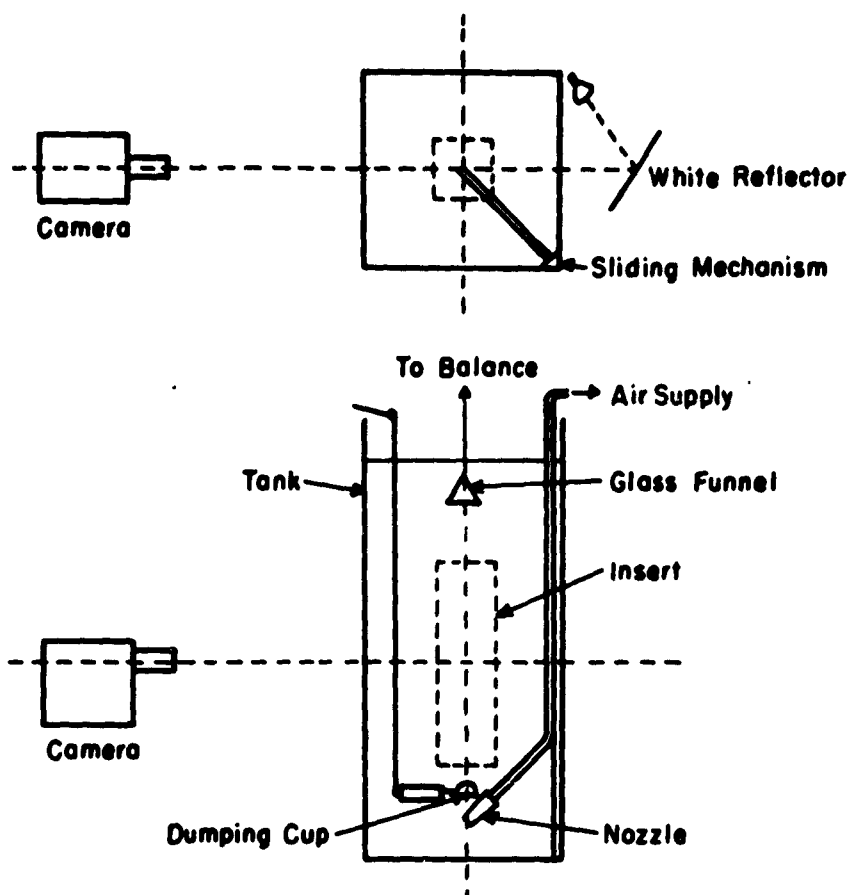


Figure 1 - Experimental Setup

BUBBLE GENERATION

Small bubbles were generated by means of hypodermic needles and glass nozzles of various sizes. The larger bubbles were obtained by use of a dumping cup, which was inverted to release the air bubble. The nozzles and needles were connected to a brass tube which was fastened to a sliding mechanism (Figure 1). This sliding mechanism allowed the tips of the various nozzles to be placed at the identical position.* The air was supplied from a compressed air bottle. A needle valve regulated the air flow so that bubbles were released at the interval desired.

DETERMINATION OF BUBBLE SIZE

The bubble size was determined by "weighing" a sufficient number of bubbles in the inverted funnel (Figure 1) by means of an analytical balance. Since the density of air is negligible in comparison to that of a liquid, the difference in balance reading equals the

*This device eliminated focusing of the camera after each change of nozzle. The camera was used to determine velocity, path, and shape of the bubble.

buoyancy of the bubbles (i.e., it equals the volume of the bubbles times the density of the liquid). The change in balance reading was always at least 0.2 gm, resulting in an accuracy of measurement of volume of 1 percent. The volume of the individual bubble was obtained by dividing the total volume by the number of bubbles collected in the funnel. A comparison of photographs of different bubbles showed that the bubble size did not vary if the frequency of bubble generation remained constant. Large bubbles from the dumping cup were weighed individually. The volume of the bubble was adjusted for the change in pressure due to difference in depth between the level at which the rate of rise is determined and the level of the inverted funnel. This was done by use of the general gas law at constant temperature, taking into account the partial pressure of the saturated vapor at test temperature.*

Tiny spherical bubbles could not be generated at a frequency to allow a sufficient number to be collected in the funnel, hence their size was determined from the photographic record. No correction for change in depth is then needed.

OTHER EXPERIMENTAL TECHNIQUES

To avoid any changes in the volume of the bubble due to air interchange with the liquid, the latter was saturated with air prior to actual testing. This was accomplished by stirring the liquid and by blowing air through it.

The temperature of the liquids, with the exception of cold and hot water, was room temperature, which varied little throughout the day. Water was cooled by circulation through a water cooler; it was heated with immersion heaters or obtained directly from the hot water faucet. Both filtered and tap water were used in the tests.

Uniformity of liquid temperature was achieved by means of mechanical stirring before each test. Frequent checks of temperature at various locations inside the tank were made by means of immersion thermometers. In the process of stirring, small bubbles appeared in the liquid. The irregularity of motion of these small bubbles, which were still present after completion of the stirring, served as an indication of the presence of residual turbulence in the liquid. In sufficient time, the motion of the small bubbles always became regular and hence indicated that the residual turbulence, if still present, was not large enough to affect the

*Details of this correction are as follows:

$$V_1 = \frac{p_0 - p'}{p_1 - p'} V_0$$

where V_1 is the volume of the bubble at the camera level,

V_0 is the volume of the bubble as determined by weighing,

p_0 is the absolute pressure at the funnel,

p_1 is the absolute pressure at the camera level, and

p' is the vapor pressure of the liquid at the test temperature.

motion of the bubbles. The actual tests were not begun until all the small bubbles reached the surface of the liquid.

The rate of bubble flow was then regulated by the needle valve so as to release bubbles with a minimum spacing of 24 in. This reduced the effect of the wake created by the passage of a bubble on the motion of a bubble following.* At higher bubble rates, the velocity of the individual bubble is increased.

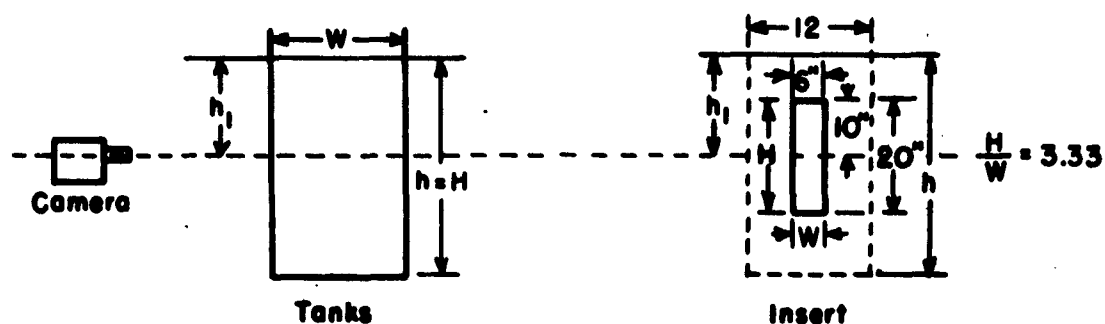
The same precaution was observed for the larger bubbles that were formed by dumping. An additional precaution was to rotate the dumping cup with steady speed in order to avoid splitting of the bubble or the formation of satellites upon release. The slow passage of the air through the brass tube inside the tank allowed the air to reach the temperature of the liquid. Contact of the air at the nozzle tip or inside the dumping cup with the liquid allowed saturation of the air with liquid vapor, so that the air bubble can, in each instance, be assumed to be saturated with the vapor of the liquid in which it rises.

MOTION PICTURES AND THEIR EVALUATION

The velocity, path, and shape of the bubbles were obtained from motion pictures made with a Mitchell 35mm camera using a special lens attachment to permit close-ups. Film speeds of 25 to 35 frames per second and back lighting from a white reflector were used. For the first few tests, the film speed was obtained by photographing a rotating clock dial; subsequently a neon timing light with a 60-cycle voltage source was utilized. The film speed was determined from the marks of the timing light on the film. The field of the camera varied from 1.4×1.8 to 1.75×2.3 in. depending upon the refractive index and horizontal depth of liquid. A transparent scale photographed in the plane of the bubble provided the distance scale factor for the evaluation of displacement and size. The camera lens was placed at approximately the midpoint between the liquid level and the bottom of the tank for all tests. The camera location was in each instance sufficiently above the nozzle tip so that the bubbles reached their terminal velocity before passing in front of the camera. A summary of camera location and depth of liquids in the tanks is given in Figure 2.

Changes in bubble volume due to differences in liquid depth were minimized by making velocity measurements over a very short vertical displacement (less than $2 \frac{1}{2}$ in.). The rate of rise of bubbles was determined by measuring the displacement of a bubble from a reference point on successive frames of the film by means of a Bausch and Lomb contour-measuring projector using a magnification of twenty-five (Figure 3). These displacements were then plotted against the frame number. The straight-line plot indicates that the velocity of the bubble remained constant during the time it passed the field of the camera. From the slope of the line, the frame speed, and the scale factor, the velocity of the bubble is computed.

*Napier²⁰ showed the absence of proximity effect for air bubbles in water, ranging in equivalent radius from 0.14 to 0.38 cm, if the frequency was below 30 bubbles per minute.



Liquid	Avg. Temp. deg C	Tank	Width of Tank W inches	Height of Liquid h inches	H/W	Height of Liquid above Camera h_1 inches
Water (Filtered)	19	Large	36	48	1.33	22.5
Water (Tap)	20	Large	36	48	1.33	25
	22	Medium	12	31.5	2.62	15.5
	20	Insert	6	36.5	3.33	20
Water (Filtered)	6	Large	36	48	1.33	23
	5	Medium	12	30	2.50	14
Water (Tap)	48	Large	36	49	1.36	23.5
	49	Medium	12	39	3.25	22.75
	48	Small	6	19	3.17	9
Glim Solution	19	Small	6	20	3.33	9.5
Mineral Oil	28	Medium	12	36	3.00	19
	27	Insert	6	36.5	3.33	19.5
Varsol	28	Large	36	39.5	1.10	24.5
	28	Medium	12	38.5	3.21	21.5
	25	Insert	6	36	3.33	19
Turpentine	23	Small	6	19.5	3.25	9
Methyl Alcohol	30	Medium	12	39	3.25	24
Water and 62 percent Corn Syrup	22	Small	6	20	3.33	9.5
Water and 68 percent Corn Syrup	21	Small	6	19	3.17	9

Figure 2 - Test Conditions

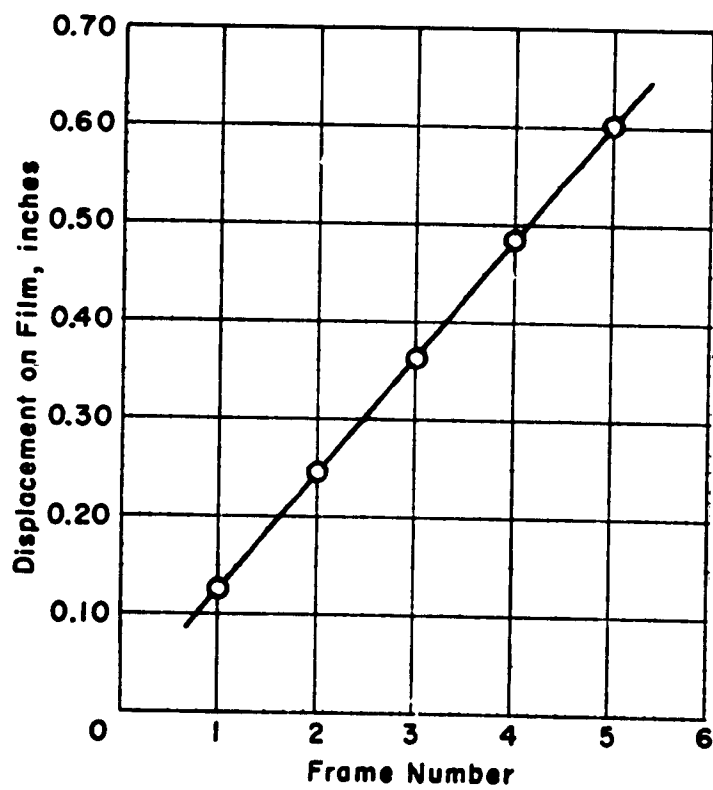
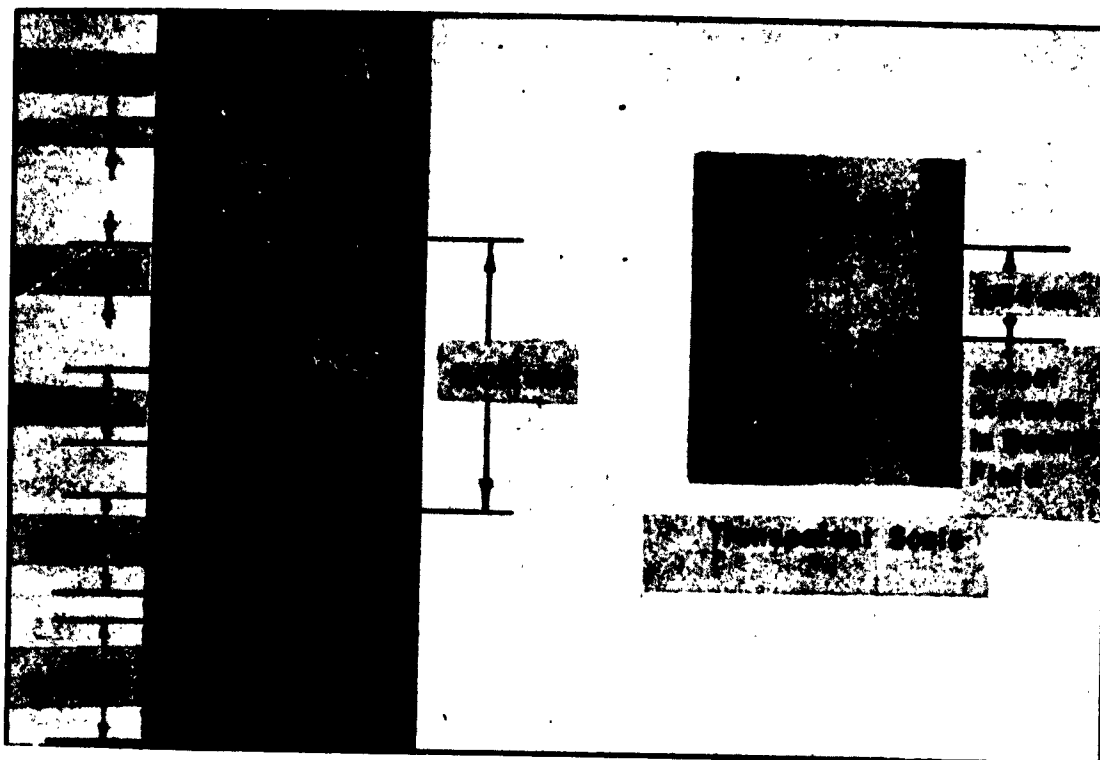


Figure 3 - Evaluation of Velocity of Rise of Bubbles

RESULTS

TERMINAL VELOCITY OF AIR BUBBLES

The results of tests to determine the velocity of rise of air bubbles in various liquids are most conveniently presented as a function of the equivalent radius of the bubble, defined as the radius of a sphere having the same volume as the bubble. Figures 4-18 show the terminal velocity of air bubbles rising freely in tap (unfiltered) and in filtered water (including data from other investigators), in water containing Glim, in mineral oil, Varsol, turpentine, methyl alcohol, and two corn syrup-water mixtures as a function of the equivalent radius. Figure 14 presents Bryn's results in an ethyl alcohol-water mixture and two glycerine-water mixtures.* Figure 15 summarizes all velocity curves (except those for tap water). A compilation of the properties of the liquids is given in Table 1 (see page 11).

In general, the results as seen from Figure 15 indicate that for small (spherical) air bubbles of given volume, the viscosity of the liquid is the most important property determining the rate of rise. Very large bubbles (spherical caps) rise independently of the properties of the liquid.

WALL AND PROXIMITY EFFECTS

As indicated previously, the effect of the container walls on the velocity of a bubble had to be determined if the results of tests conducted in a tank of limited dimensions were to be applied to bubble motion in an infinite medium. Tests were, therefore, conducted in tanks of different sizes in water, Varsol, and mineral oil. Figure 4 gives the results of tests conducted with filtered water by several investigators including the Taylor Model Basin. The cross sections of the containers used are also indicated in the figure. No wall effect is noticeable from these results. For example, Gorodetskaya's results for bubbles ranging from 0.01 to 0.07 cm rising in a tube of 5 cm diameter show no wall effect when compared with results of tests conducted in larger containers. The results of the present experiments, given in Figures 5, 6, 7, 9, and 10, show within experimental accuracy, the absence of any wall effect for the range of bubble sizes tested. Subsequent tests in the other liquids were made in the small tank only and the results may be applied to the case of an infinite medium.

No systematic investigation was made of vertical proximity effect, i.e., the effect of the wake created by the passage of a bubble on the motion of a bubble rising at a distance

*The results of the 81 percent (by weight) glycerine-water mixture have been omitted. Bryn presented these results in terms of drag coefficient and Reynolds number, from which the terminal velocity can be computed. In the region of bubble size where the rate of rise is shown to be a function of size only (hence a common velocity curve for all liquids; see Figure 15), the velocity curve for the 81 percent mixture falls appreciably above the common curve. The discrepancy is probably due to erroneous evaluation of the two dimensionless parameters for the 81 percent glycerine-water mixture.

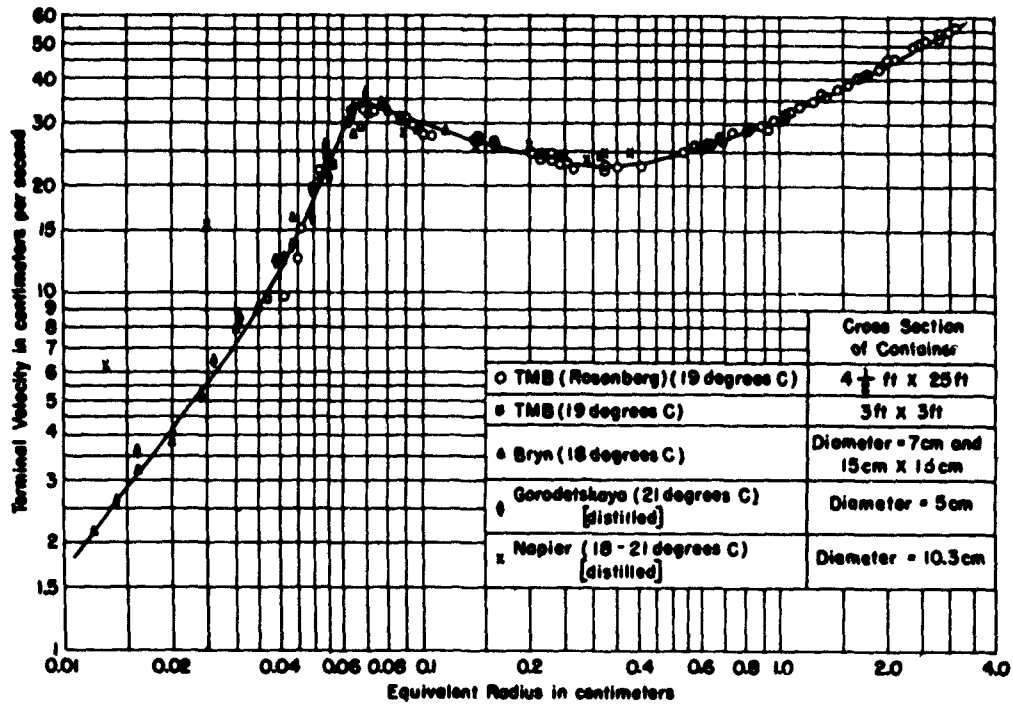


Figure 4 - Terminal Velocity of Air Bubbles in Filtered or Distilled Water as a Function of Bubble Size

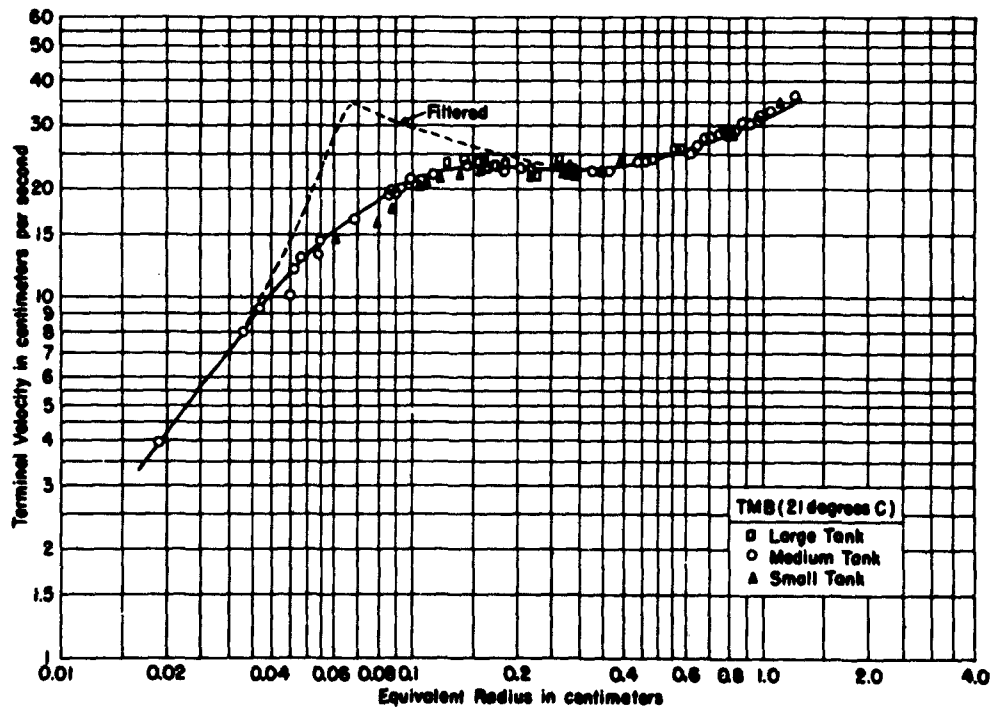


Figure 5 - Terminal Velocity of Air Bubbles in Tap Water as a Function of Bubble Size

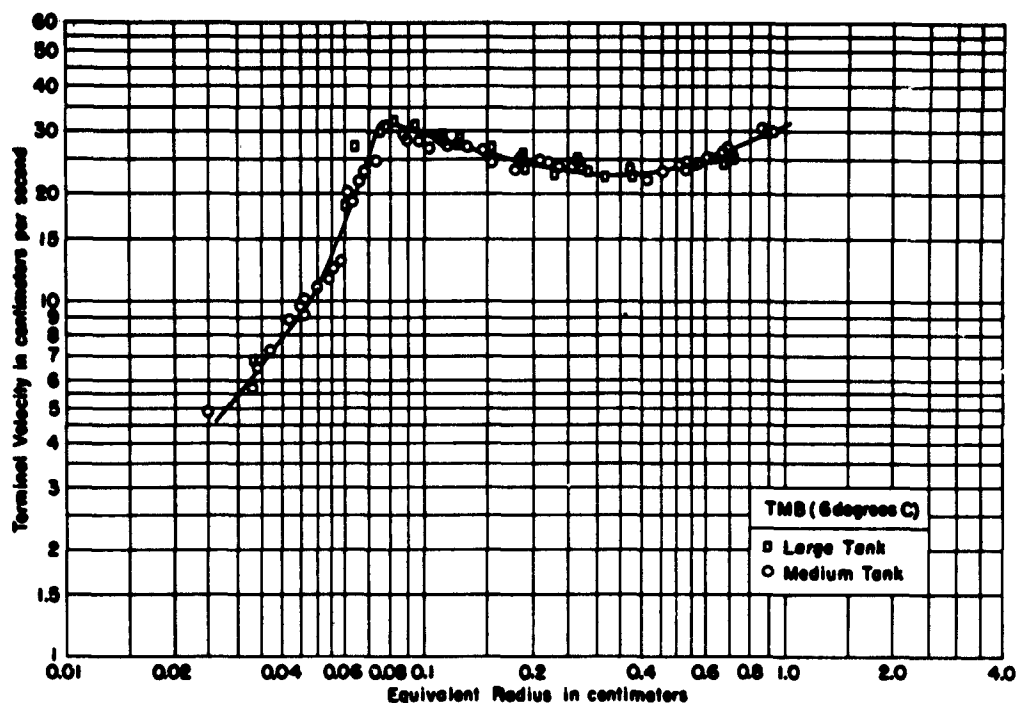


Figure 6 - Terminal Velocity of Air Bubbles in Cold Filtered Water as a Function of Bubble Size

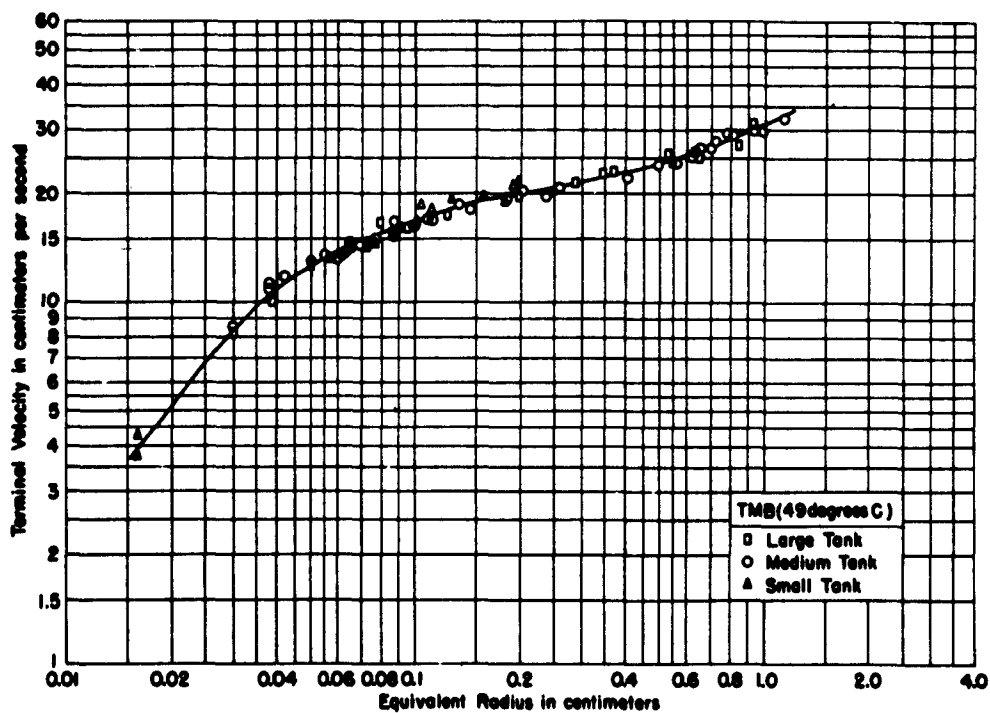


Figure 7 - Terminal Velocity of Air Bubbles in Hot Tap Water as a Function of Bubble Size

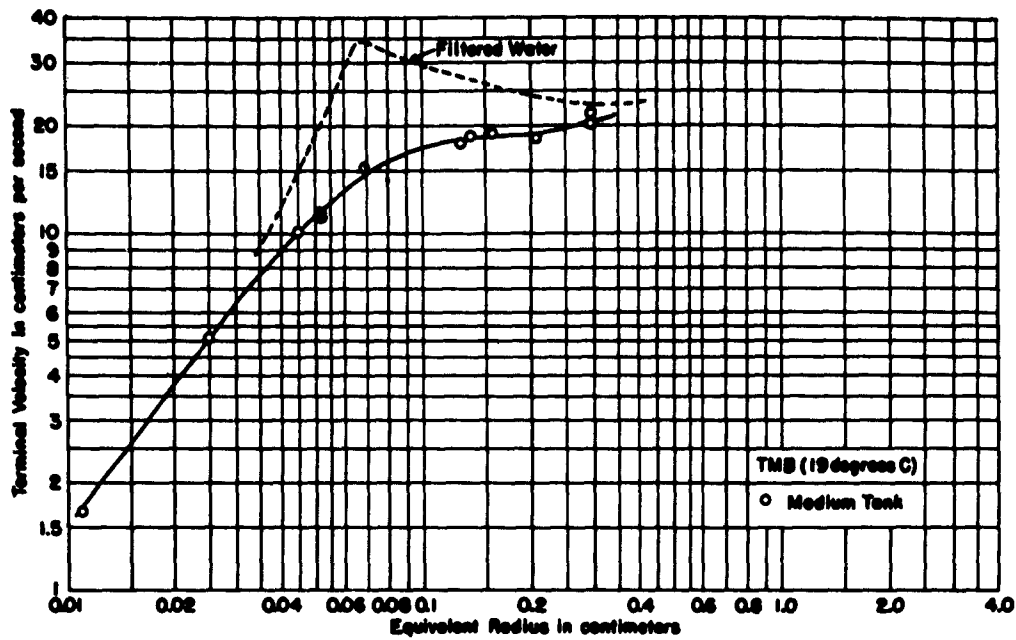


Figure 8 - Terminal Velocity of Air Bubbles in Water Containing Glim as a Function of Bubble Size

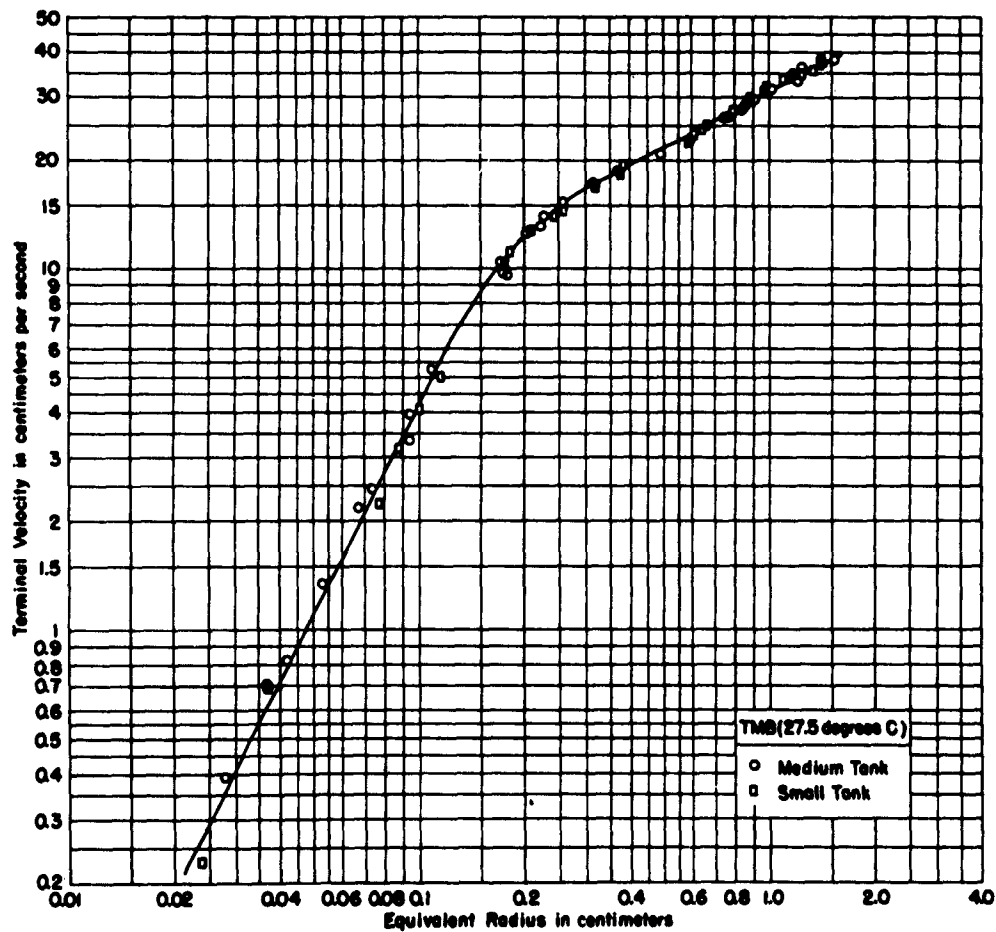


Figure 9 - Terminal Velocity of Air Bubbles in Mineral Oil as a Function of Bubble Size

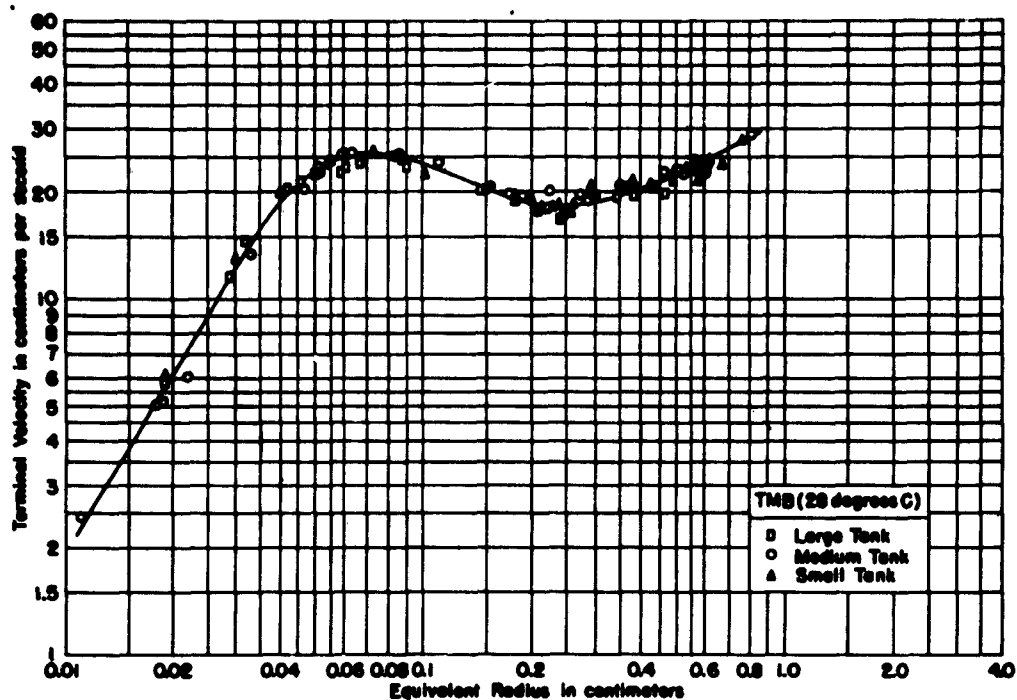


Figure 10 - Terminal Velocity of Air Bubbles in Varsol
as a Function of Bubble Size

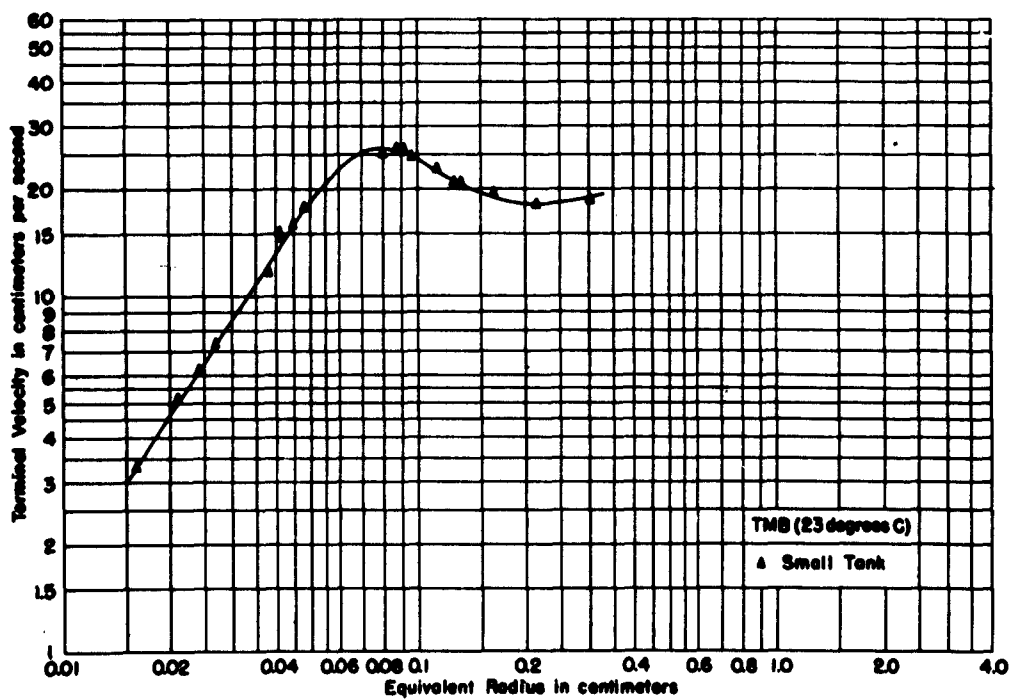


Figure 11 - Terminal Velocity of Air Bubbles in Turpentine
as a Function of Bubble Size

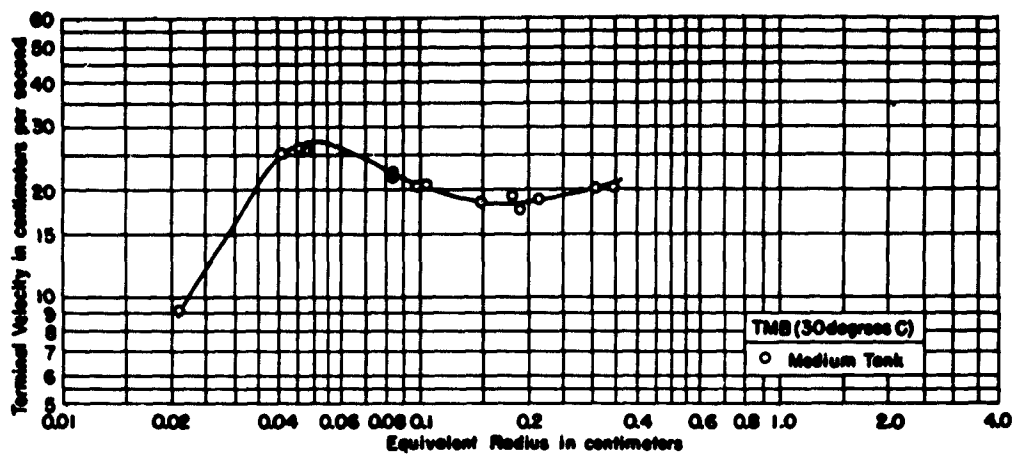


Figure 12 - Terminal Velocity of Air Bubbles in Methyl Alcohol as a Function of Bubble Size

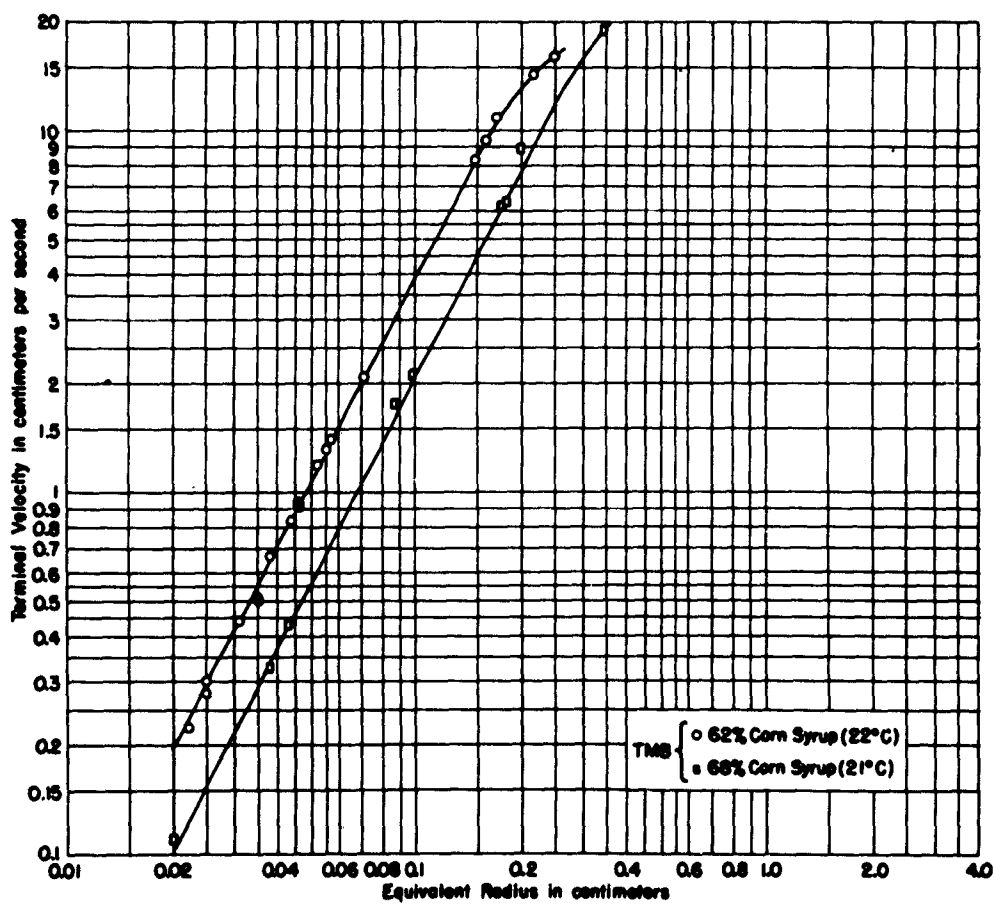


Figure 13 - Terminal Velocity of Air Bubbles in Corn Syrup-Water Mixtures as a Function of Bubble Size

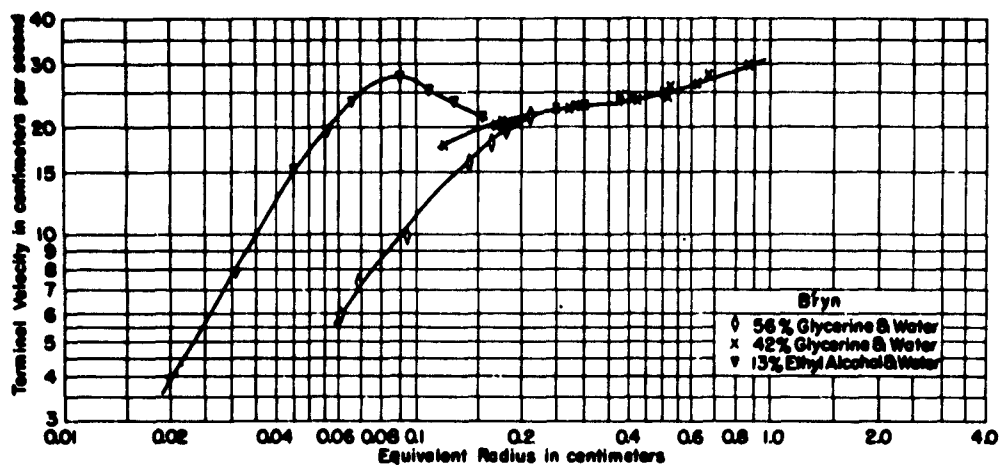


Figure 14 - Terminal Velocity of Air Bubbles in Liquids as Obtained from Bryn's Data

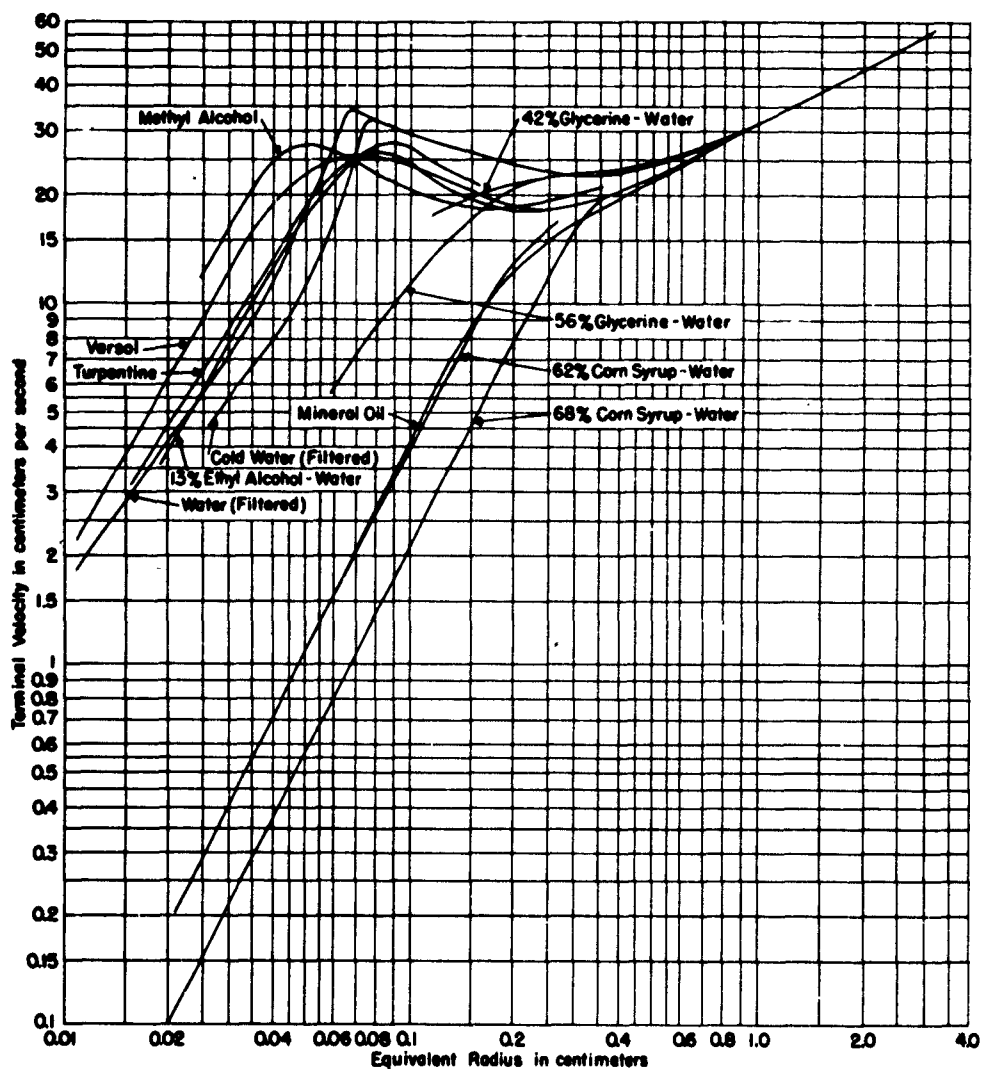


Figure 15 - Comparison of Terminal Velocities of Air Bubbles in Various Liquids

below. Such effects were avoided in the experiments by sufficient spacing between the bubbles. However, the results of a few special observations indicate that proximity effects may be appreciable. For example, tests in mineral oil show an increase of 9 percent and 39 percent for bubbles of equivalent radius of 0.17 cm, rising 7.7 cm and 3.2 cm apart, respectively. Napier²⁰ observed an increase of 6 percent for bubbles of 0.14 cm radius in water, rising 6 cm apart. The presence of the wake in the liquid thus results in higher velocities of rise of the bubble.

NONDIMENSIONAL PRESENTATION OF BUBBLE DATA

In a previous section it was pointed out that presentation of the experimental data on air bubbles in terms of dimensionless products gives complete correlation provided the variables considered in the analysis are complete and pertinent. The results of the Taylor Model Basin bubble tests and those of Arnold,¹⁰ Bond and Newton,¹¹ and Bryn²⁴ are given in terms of the drag coefficient, Reynolds number and the parameter M in Figure 16.* Figure 17 presents the bubble data in terms of the drag coefficient, Weber number, and the parameter M . The curve for filtered or distilled water at a temperature of 19 deg C was drawn through points obtained from the experiments of Bryn and the Taylor Model Basin tests.

Examination of Figure 16 or 17 shows no systematic arrangement of the curves with change in the parameter M , which is constant for a specific liquid. It can, therefore, be concluded that neither of the nondimensional sets presented nor any other complete set using the same six variables (namely, velocity, acceleration of gravity, density and viscosity of the liquid, surface tension, and equivalent radius) is sufficient for a complete description of bubble motion.

The question now arises whether correlation of bubble data could be obtained by using two additional dimensionless parameters, for example, the liquid to air viscosity and density ratios or the Reynolds number inside the bubble and the density ratio, etc. The results of the experiments conducted do not permit conclusions regarding the importance of these parameters. A short discussion of the significance of the internal Reynolds number will be given in subsequent sections.

SPHERICAL BUBBLES

It was observed in the experiments that, as the bubble size was increased, a change in bubble shape from spherical to ellipsoidal to spherical cap shape occurred in all liquids. Very small bubbles are spherical. Larger bubbles are flattened, i.e., ellipsoidal in shape, whereas very large bubbles assume a spherical cap shape. Of course, the volumes at which

*The results for tap water and for water containing Glim are not shown. They will be discussed in subsequent sections.

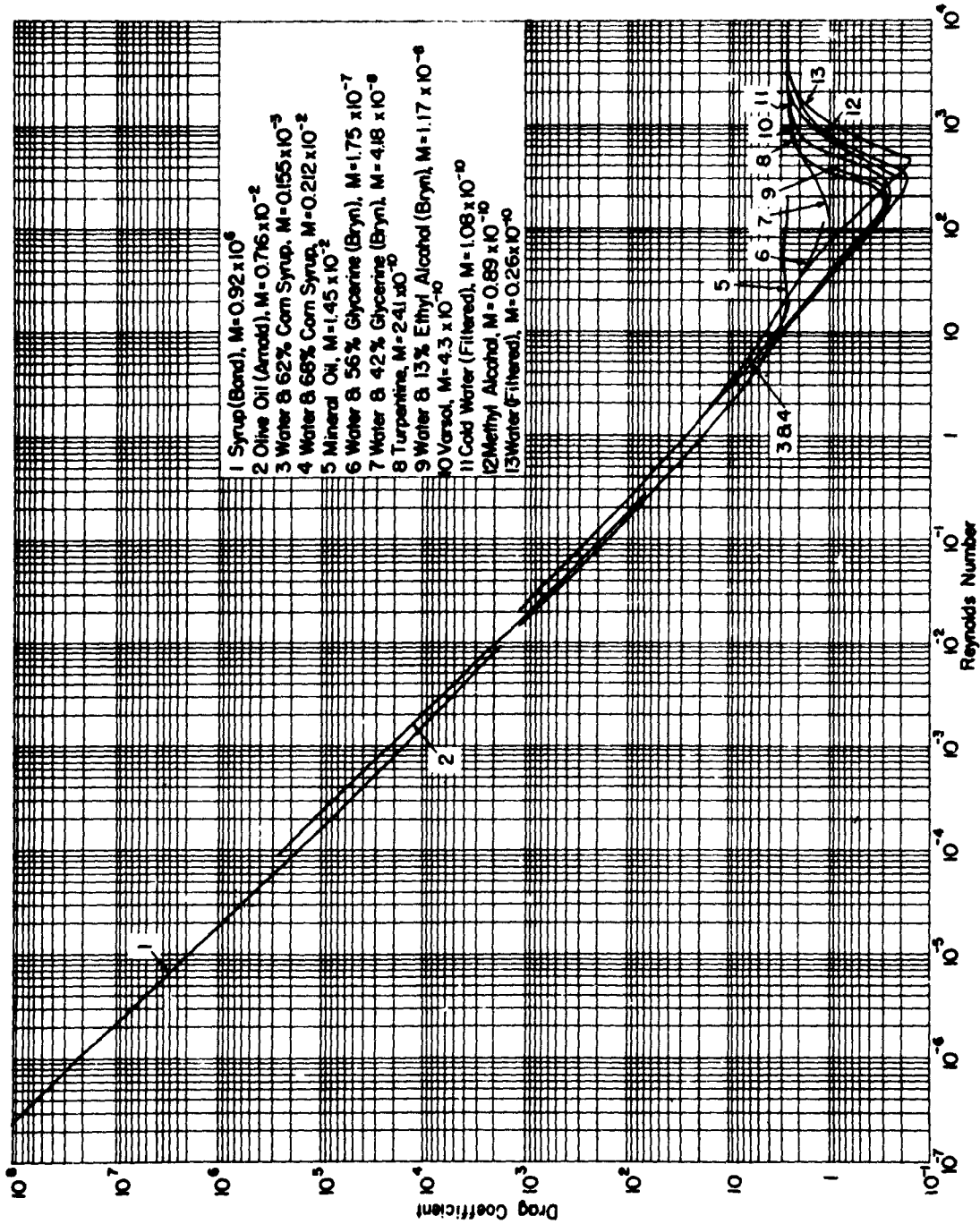


Figure 16 - Drag Coefficient as a Function of Reynolds Number for Air Bubbles Rising at Their Terminal Velocity in Various Liquids

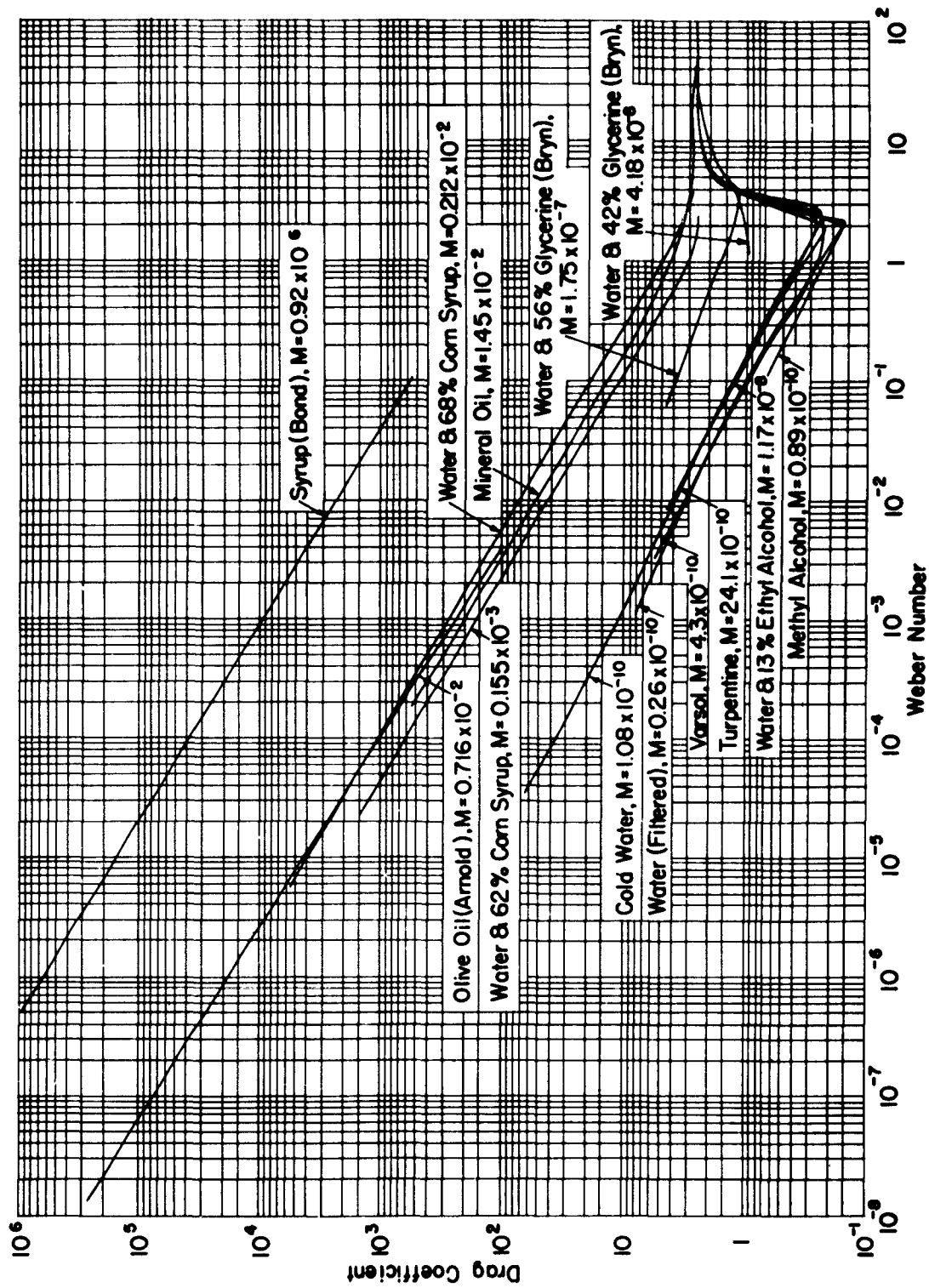


Figure 17 - Drag Coefficient as a Function of Weber Number for Air Bubbles Rising at Their Terminal Velocity in Various Liquids

these transitions occur vary with the liquids. Photographs of typical shapes are shown in Figure 18. It should be noted here that some of the shapes shown in these photographs are instantaneous shapes, since the shape of large bubbles does not remain constant during the ascent. An exception are bubbles rising in a highly viscous medium (e.g., mineral oil and corn syrup).

The results for the spherical bubbles only are plotted in terms of the drag coefficient and the Reynolds number in Figure 19, with Reynolds numbers ranging up to about 400. The drag curve for rigid spheres is also included.⁴⁸ From it, the following can be observed: The drag curves of spherical bubbles in the various liquids fall between two limiting curves. As upper limit, the drag curve of rigid spheres is obtained, while the lower limit is the drag curve for fluid spheres. With decreasing Reynolds number, the rigid sphere curve connects with the straight line of Stokes' Law, while the fluid sphere curve connects with the line of Hadamard-Rybczynski's Law. The curve for the fluid spheres was obtained by drawing the lower envelope to the experimental curve; its accuracy can be confirmed by additional tests in other liquids or by extension of the theoretical solution into regions beyond that of very slow flow.

It will be noted from Figure 19 that the curve for mineral oil, for example, follows the straight line of Hadamard-Rybczynski's Law over a certain region of Reynolds numbers. This indicates that the boundary conditions assumed in the analytical solution for fluid spheres are actually fulfilled and that circulation exists inside the bubble. Circulation inside bubbles has been observed experimentally.⁴⁹

The experimental curves of Figure 19 also indicate an interesting aspect of the phenomenon of bubble motion, namely that with decreasing Reynolds number, the drag coefficient of the bubbles becomes equal to the drag of rigid spheres. This transition may occur at a Reynolds number of about 40 (as for filtered and distilled water) or may not take place until very low Reynolds numbers are reached, i.e., well within the region of slow flow (as for olive oil¹⁰ or very viscous syrup¹²). Thus, from the experimental data available, it appears certain that tiny air bubbles rising in any liquid follow Stokes' Law.

For bubbles behaving like rigid bodies, thus indicating absence of motion inside the bubble, the internal Reynolds number (although nonvanishing) is of no significance in describing the rising motion of the bubbles. Likewise, the internal Reynolds number cannot be used to predict the transition point at which the drag of the bubbles becomes less than that of corresponding rigid spheres. Beyond this transition point, the internal Reynolds number might be of importance in describing the motion of the bubbles.

Surface tension tends to make the surface area of the bubble as small as possible. For a given volume, the configuration of minimum surface area is a sphere. This effect of surface tension would be most pronounced for bubbles of small radii.

Liquid	Approximate Equivalent Radius, cm				
	0.02	0.03	0.05	0.07	0.1
Methyl Alcohol			.	.	.
Varsol	
Turpentine	
Water Filtered 6 deg C	
Water Filtered 19 deg C	
Mineral Oil	
62 percent Corn Syrup and Water	
68 percent Corn Syrup and Water	
Water, Tap 21 deg C	
Water, Tap 49 deg C	
Water and 0.42 percent Glim 19 deg C	

Figure 18a

ELLIPSOIDAL BUBBLES

For larger bubble sizes, the surface forces, which are essential in maintaining the spherical shape of a bubble, become smaller in comparison to the viscous and hydrodynamic forces, and flattening of the bubble occurs. This flattening to approximately an oblate spheroid results in higher drag as compared to a sphere of the same volume. Figure 20 shows the drag curves of ellipsoidal and spherical cap bubbles in terms of the Reynolds number. The estimated extent of the regions of ellipsoidal and spherical cap bubbles is indicated in the figure. It will be observed that the region of ellipsoidal bubbles for the various liquids













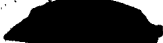


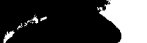












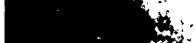








Approximate Equivalent Radius, cm					Liquid
0.2	0.3	0.5	0.7	1.0	
					Methyl Alcohol
					Varsol
					Turpentine
					Water Filtered 6 deg C
					Water Filtered 19 deg C
					Mineral Oil
					62 percent Corn Syrup and Water
					68 percent Corn Syrup and Water
					Water, Tap 21 deg C
					Water, Tap 49 deg C
					Water and 0.42 percent Glim 19 deg C

Figure 18b

Figure 18 - Typical Shapes of Bubbles of Several Volumes in the Various Liquids

occurs at different ranges of Reynolds number, that for liquids of low " M " number a minimum in the drag curves is reached at Reynolds numbers of the order of 250, and that these minima occur near the transition from spherical to ellipsoidal shape. Such minima are not obtained for liquids of high " M " number. The drag coefficients of bubbles in such liquids decrease until a constant value for the drag coefficient (spherical caps) is attained. In the ellipsoidal region the curves are arranged according to the magnitude of the " M " number, indicating that the liquid properties contained in this parameter, namely surface tension, viscosity, and

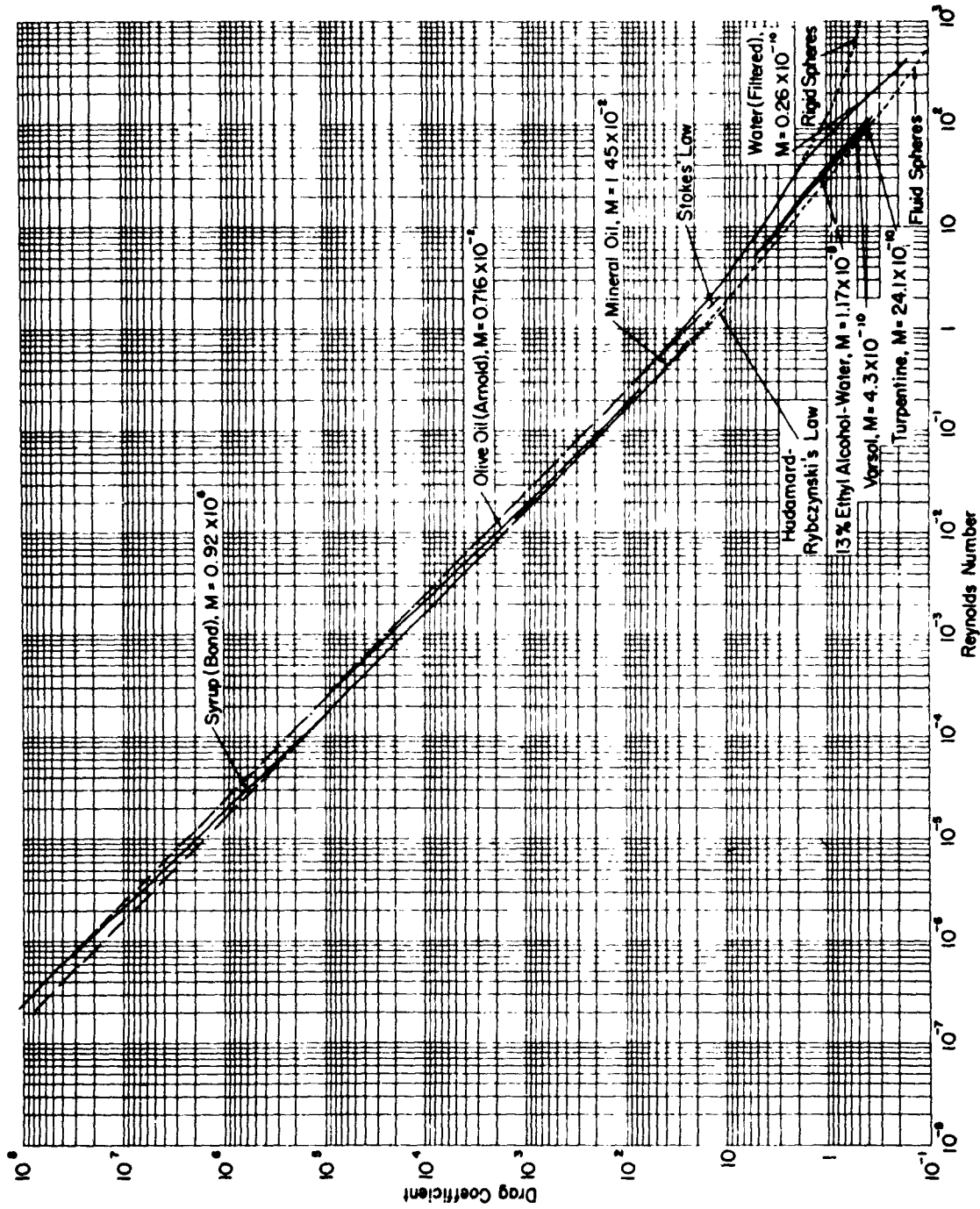


Figure 19 - Drag Coefficient as a Function of Reynolds Number for Spherical Air Bubbles Rising at Their Terminal Velocity in Various Liquids

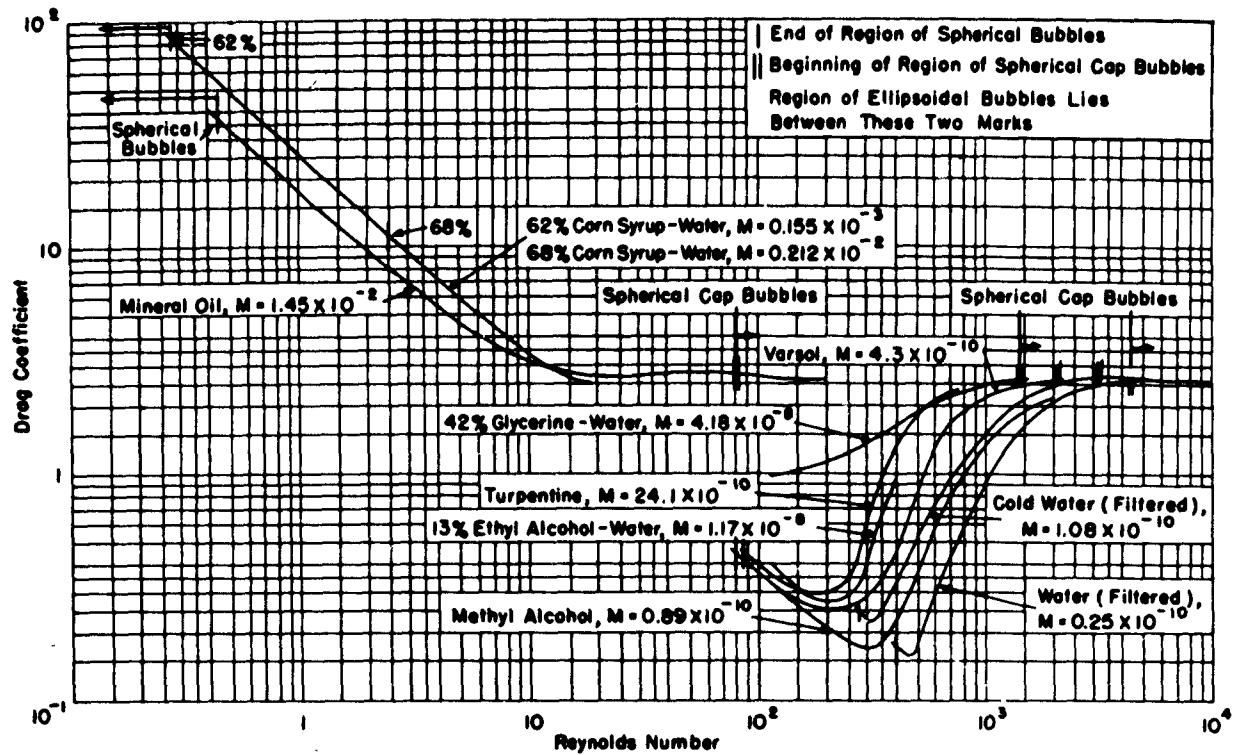


Figure 20 - Drag Coefficient as a Function of Reynolds Number for Ellipsoidal and Spherical Cap Bubbles in Various Liquids

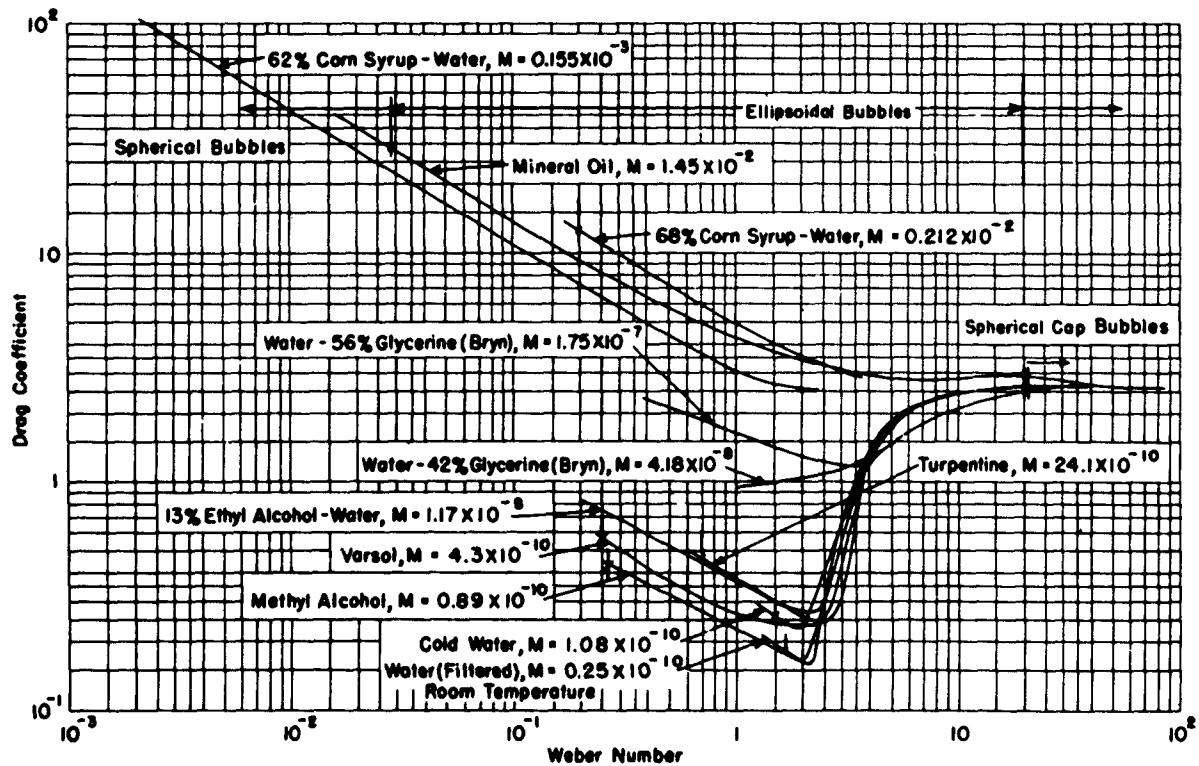


Figure 21 - Drag Coefficient as a Function of Weber Number for Ellipsoidal and Spherical Cap Bubbles in Various Liquids

density, are of primary importance in the motion of these bubbles.

Thus, in this region the Reynolds number inside the bubble is of no importance in the description of bubble rise, since correlation was obtained in terms of drag coefficient, Reynolds number, and "M" number.

In Figure 21 the drag coefficient of the bubbles was plotted as a function of Weber number. It is seen that, for the liquids tested, transition to a constant value of drag coefficient (spherical cap region) is reached at a Weber number of about 20.

SPHERICAL CAP BUBBLES

When the viscous and surface tension forces become small relative to the hydrodynamic forces, the shape assumed by the bubbles is that of the so-called spherical caps. Typical shapes of these bubbles are shown in Figure 18. The upper surface is essentially spherical, while the lower surface varies from a highly irregular one for liquids of low viscosity to a smooth surface for very viscous liquids. The configuration of the upper surface results almost exclusively from the hydrodynamic forces.*

The geometric similarity of these bubbles was shown by Rosenberg¹ and Davies and Taylor,²² who determined a constant drag coefficient of 2.6 for them. The results of the present tests in a number of liquids confirm this value; Figure 21. The velocity of spherical cap bubbles of given size rising in any liquid can be determined from the constant value of the drag coefficient or directly from the velocity curve (Figure 15). For $C_D = (8/3)gr_c/U^2 = 2.6$, we obtain for the rate of rise of the spherical caps in all liquids

$$U = 1.02\sqrt{gr_c}$$

Thus, the velocity of rise of these bubbles is a function of the bubble size only and not of the properties of the liquid (see special case of "Dimensional Analysis").

PATH OF BUBBLES

Figures 22-25 show representative paths and corresponding shapes of bubbles in the various liquids. Three types of motion of the bubbles were observed in the experiments: (1) rectilinear motion, (2) motion in a helical path, and (3) rectilinear motion with rocking. The motion of spherical bubbles is either rectilinear or helical. For ellipsoidal and spherical cap bubbles, all three types of motion can occur. It appears that the type of motion may be predicted from the value of the Reynolds number at which the motion takes place. Below Reynolds numbers of about 300 the motion is rectilinear. With increase in Reynolds number spiraling begins and increases in amplitude and frequency until a maximum is reached. At Reynolds numbers of about 3000, the spiraling disappears and only rectilinear motion with

*See, e.g., Reference 22

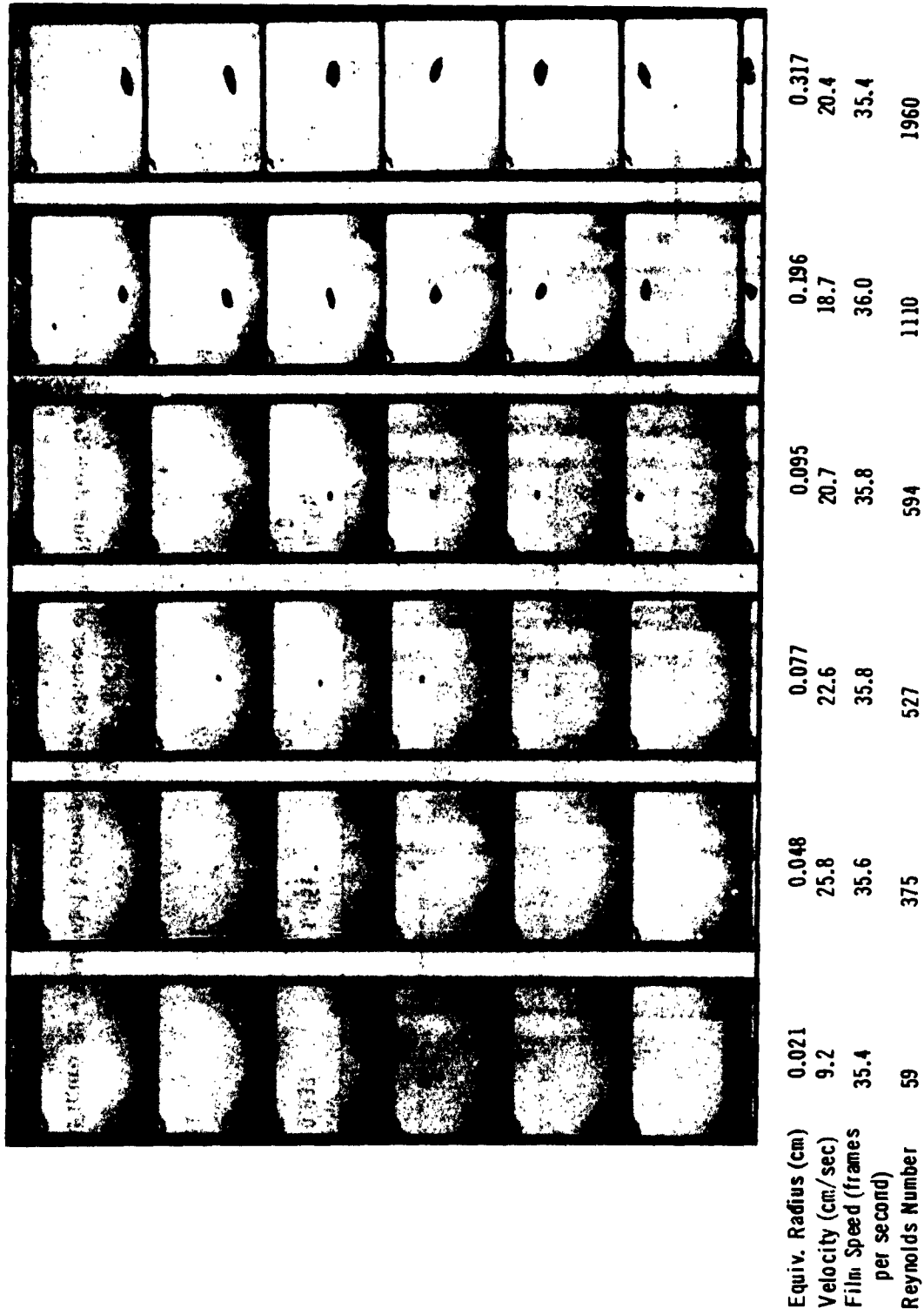
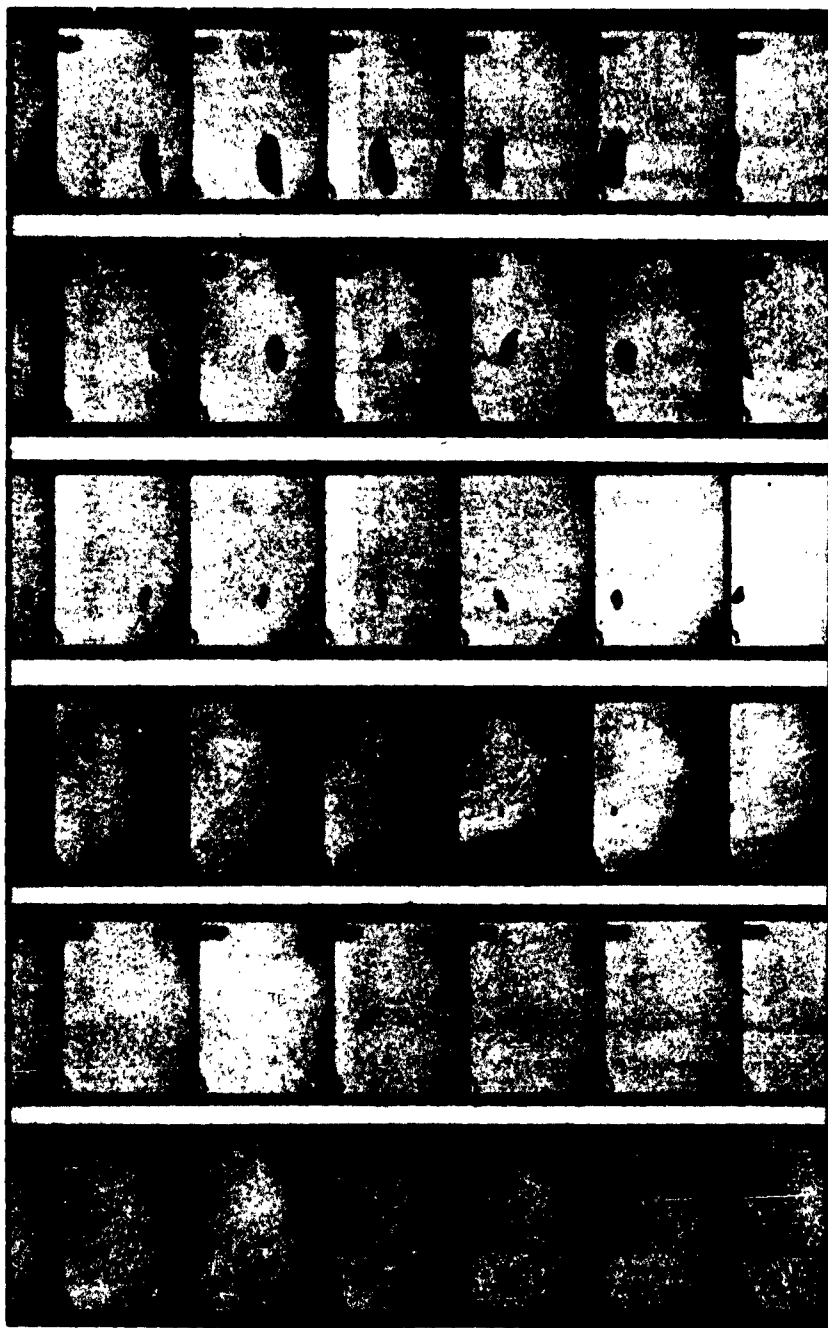
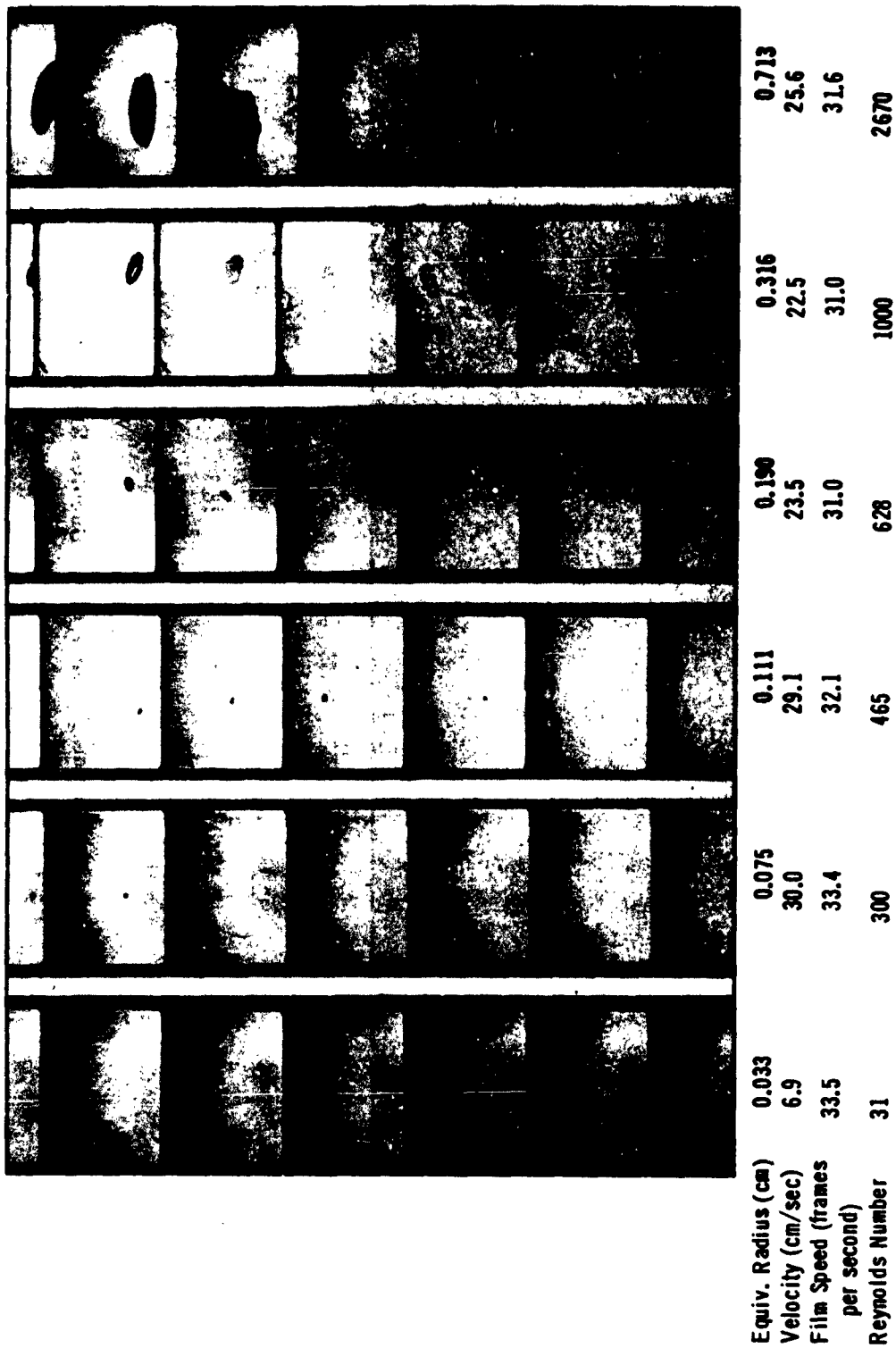


Figure 22 - Path and Corresponding Shapes of Air Bubbles Rising in Methyl Alcohol



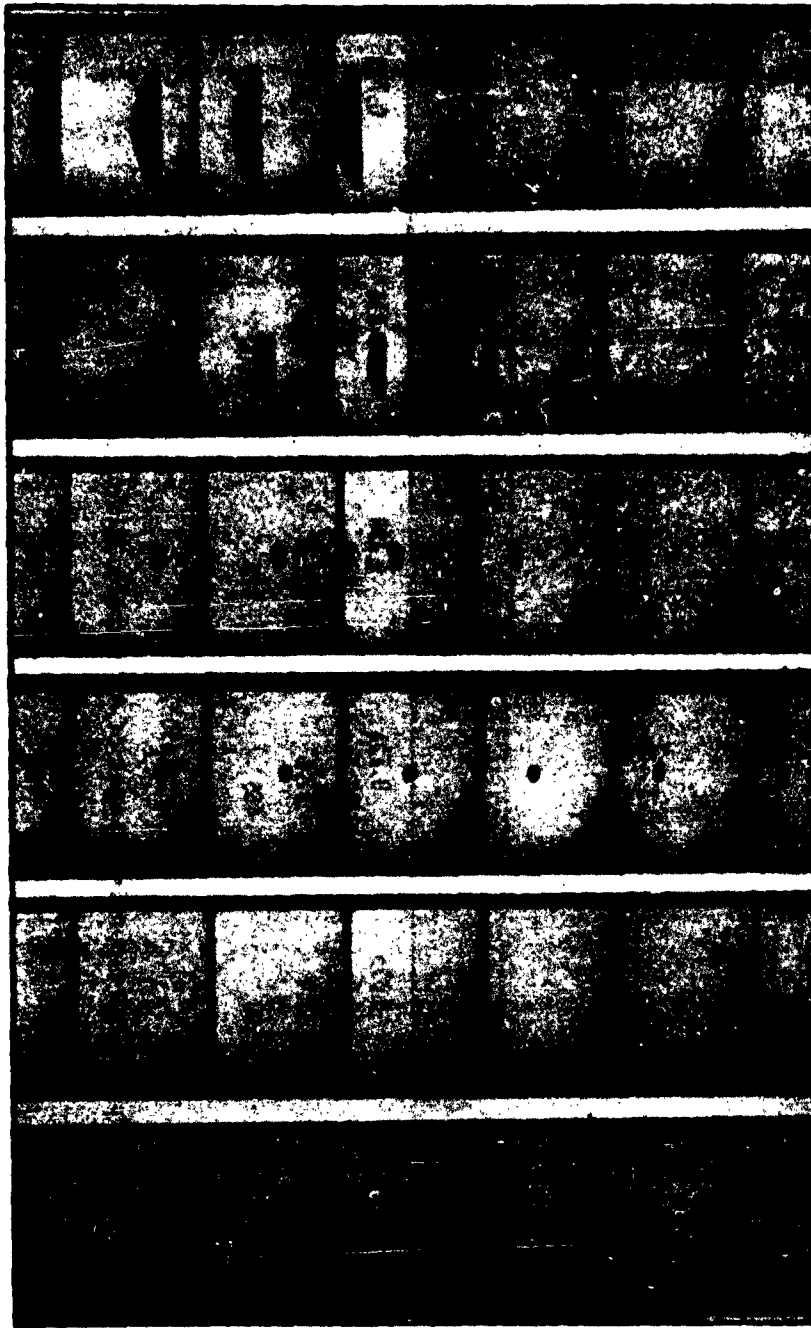
Equiv. Radius (cm)	0.033	0.051	0.103	0.206	0.322	0.483
Velocity (cm/sec)	13.3	22.5	24.2	20.0	20.8	23.1
Film Speed (frames per second)	35.9	35.6	36.0	36.0	35.7	35.6
Reynolds Number	81	212	462	762	1240	2060

Scale: 0.431/1
Figure 23 - Path and Corresponding Shapes of Air Bubbles Rising in Varsol



Scale: 0.490/1'

Figure 24 - Path and Corresponding Shapes of Air Bubbles Rising in Cold Water



Equiv. Radius (cm)	0.037	0.074	0.199	0.302	0.570	0.987
Velocity (cm/sec)	0.688	2.45	12.7	17.9	22.7	31.5
Film Speed (frames per second)	35.5	32.3	35.6	35.5	30.1	29.1
Reynolds Number	0.080	0.572	8.05	18.3	41.7	100

Scale: 0.464/1

Figure 25 - Path and Corresponding Shapes of Air Bubbles Rising in Mineral Oil

rocking is obtained. For the bubbles rising in mineral oil and the corn syrup mixtures, only rectilinear motion without rocking was observed. The maximum Reynolds number reached during those tests was 150.

The helical path of the bubbles assumes either a clockwise or counterclockwise direction, depending upon conditions at generation. The velocity of rise of these bubbles is not affected by the sense in which the bubble revolves. The major axis of ellipsoidal bubbles is always perpendicular to the direction of motion.

The oscillatory motion of bubbles is probably caused by the periodic shedding of vortices behind the bubble. Such vortex shedding has been observed experimentally for rigid spheres at the same magnitudes of Reynolds numbers as for bubbles.⁵⁰

BUBBLES IN FILTERED AND TAP WATER

The results of the experiments in filtered water at room temperature and in cold filtered water are presented in terms of drag coefficient and Reynolds number in Figure 26. The results of Bryn,²⁴ Allen,⁹ and Gorodetskaya³⁰ are also included. It is seen that the drag curves at the two different temperatures coincide in the spherical and spherical cap region. In the region of ellipsoidal bubbles, the drag coefficient at a given Reynolds number increases with increase in "*M*" number. Gorodetskaya and Allen conducted their experiments in distilled water. A comparison of their experimental data with those obtained in filtered water shows, within experimental accuracy, no difference in the drag of air bubbles rising in filtered and distilled water.

The drag coefficients for air bubbles rising in tap water at two different temperatures are given in Figure 27. Gorodetskaya's results in tap water at room temperature are included in the figure. For comparison, the drag curves for bubbles in filtered water at room temperature and for rigid spheres are also shown. Again, in the region of spherical and spherical cap bubbles, the drag curves at the two temperatures coincide. The value of the minimum drag coefficient is, however, greater than that of the corresponding filtered water. In general, for Reynolds numbers up to about 300, the drag curves of bubbles in tap water follow closely the curve of rigid spheres.*

Thus, the results of the experiments (given in Figures 5, 26, and 27) show that for bubbles (ranging in equivalent radius from 0.035 to 0.25 cm) it is important whether the motion occurs in filtered (distilled) or tap water.

In view of the fact that merely filtering the water was sufficient to produce a change in the drag of the bubbles, it is indicated that the presence of minute particles causes this change. Minute particles, most of which can be removed by filtering, are known to exist in ordinary tap water. Specifically, if such particles are present in the water a high concentration

*It should be noted here that the physical properties of tap water did not differ from those of filtered water.

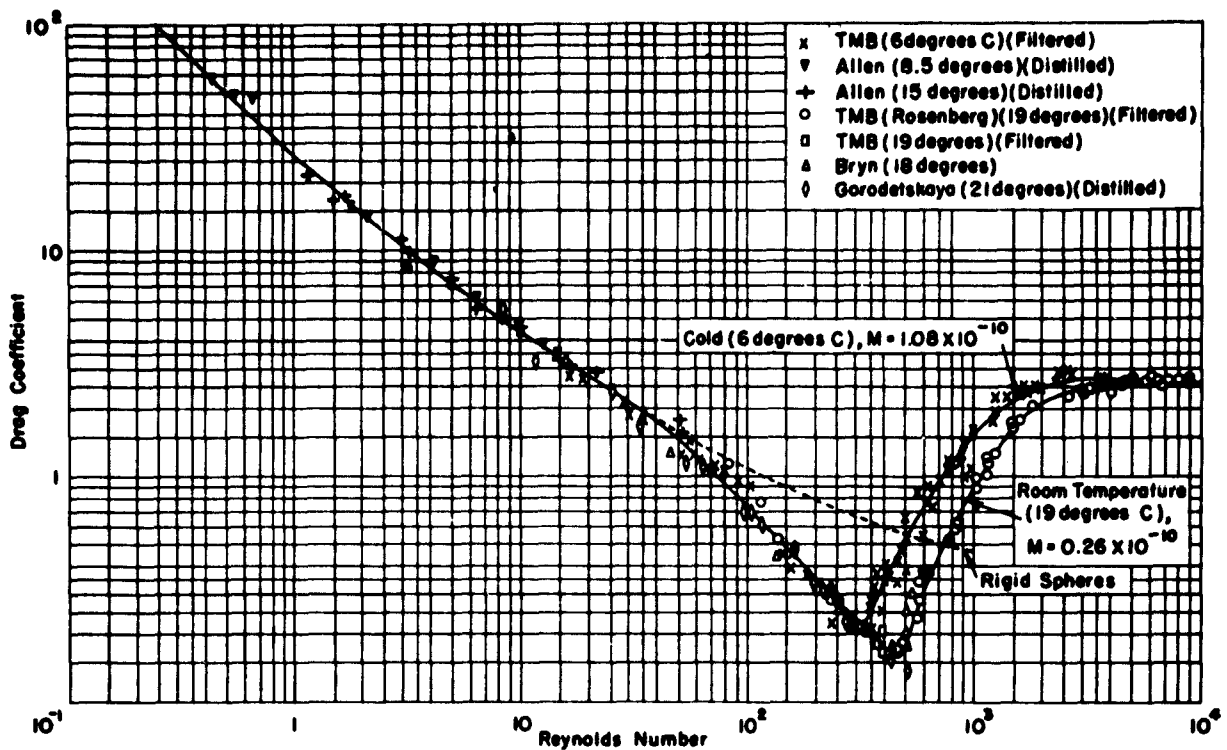


Figure 26 - Drag Coefficient as a Function of Reynolds Number for Air Bubbles Rising at Their Terminal Velocity in Filtered or Distilled Water at Different Temperatures

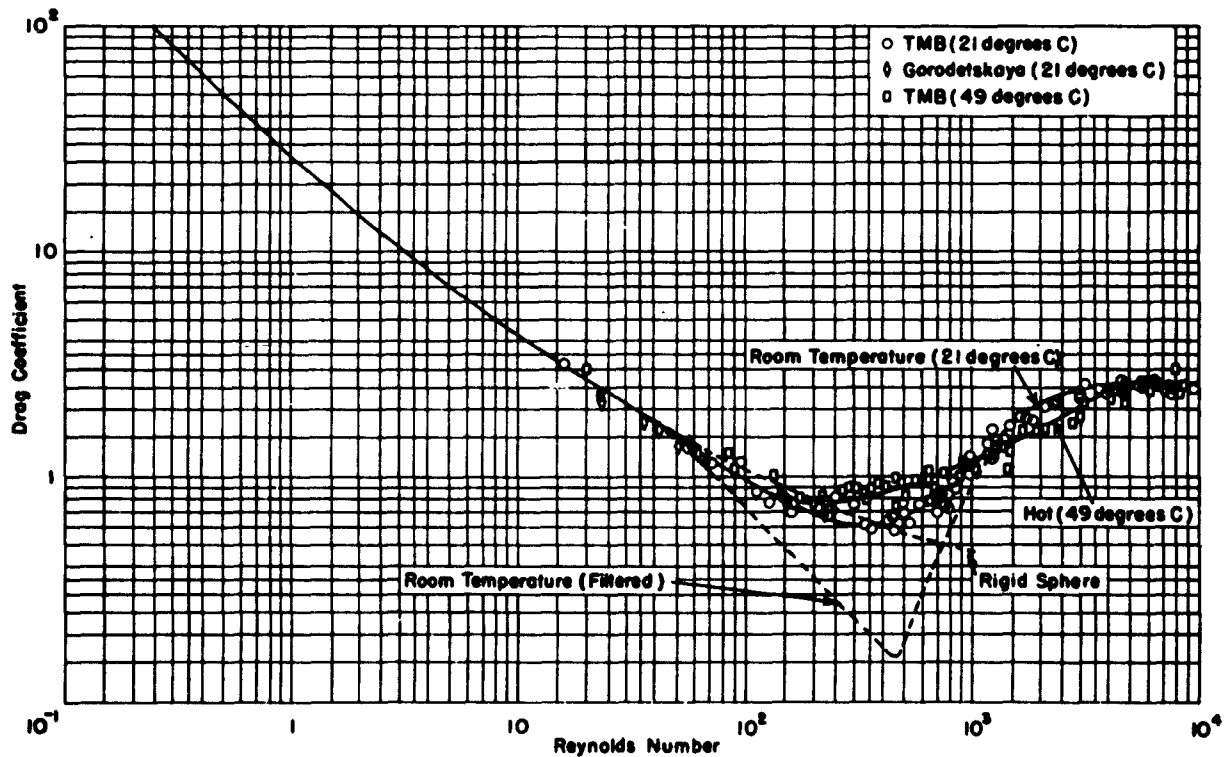


Figure 27 - Drag Coefficient as a Function of Reynolds Number for Air Bubbles Rising at Their Terminal Velocity in Tap Water at Two Different Temperatures

of these particles would be found at the surface of the bubble. The particles at the surface would travel with the bubble, hence imparting, in effect, a rigid surface to the bubble. With increasing bubble velocities, the shear forces become large in comparison to the forces holding the particles to the surface and hence at a certain critical velocity no high concentration of particles on the surface can exist.

Figure 28 compares the drag curve of bubbles in tap water obtained in the Model Basin study and the experimental data of air bubbles rising in water at room temperature as obtained by various other investigators. Reports of these other studies give no information regarding the type (tap, filtered, or distilled) of water used, but presumably these experiments were also conducted in tap water.

EFFECT OF SURFACE-ACTIVE SUBSTANCES

The effect of surface-active substances on the rate of rise of air bubbles has previously been investigated by Gorodetskaya,³⁰ who added small concentrations of various alcohols to water and concluded that, beyond a certain critical concentration of the surface-active substance, the rate of rise of the air bubbles is not affected. Stuke⁴⁰ ran experiments with oxygen bubbles rising in water containing small concentrations of caproic acid. The concentrations of the alcohols and the caproic acid were relatively small, hence the decrease in the surface tension was only about 1 dyne/cm. In the present study, the authors conducted tests in water containing Glim, a liquid detergent. The concentration of Glim (0.42 percent by volume) was high enough to decrease the surface tension by 40 dynes/cm. No measurable change in the viscosity and density of the test liquid due to the presence of Glim was noted (see Table 1). This was also true for the alcohol and caproic acid solutions. Results from these experiments are presented in terms of the drag coefficient and Reynolds number in Figure 29.*

The drag curve for bubbles in the Glim solution, as well as the experimental data from the other investigators in water containing at least the critical concentration of the surface-active substance, follows the drag curve of rigid spheres to a Reynolds number of about 200.** In the region of Reynolds numbers of 10 to 200, the drag curve for bubbles rising in a pure liquid having an "M" number very close to that of the Glim solution*** follows the drag curve of fluid spheres. Thus, the motion of bubbles in water containing surface-active materials cannot be compared with that of bubbles in pure liquids on the basis of drag coefficient, Reynolds number, and "M" number, even in the region of ellipsoidal bubbles. Although the

*As shown in the Appendix, no significant difference in the terminal velocity of oxygen and air bubbles rising in distilled water is obtained. Hence, inclusion of the results of the tests with oxygen bubbles in water containing surface-active substances is justified.

**For concentrations below the critical, the drag curve lies between that of pure water and the curve shown in Figure 29.⁴⁰

***The "M" number of Glim was 2.78×10^{-10} , that of Varsol, for example, was 4.3×10^{-10} .

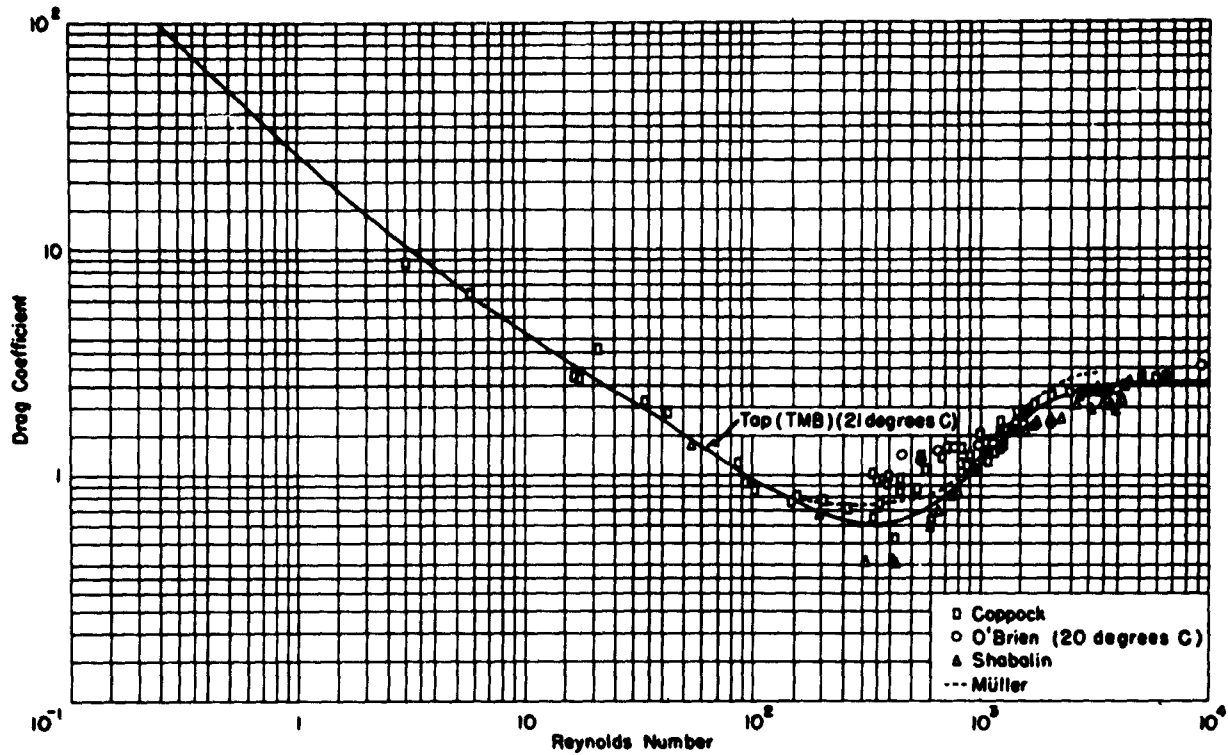


Figure 28 - Drag Coefficient as a Function of Reynolds Number for Air Bubbles Rising at Their Terminal Velocity in Water at Room Temperature as Obtained from Data of Various Investigators

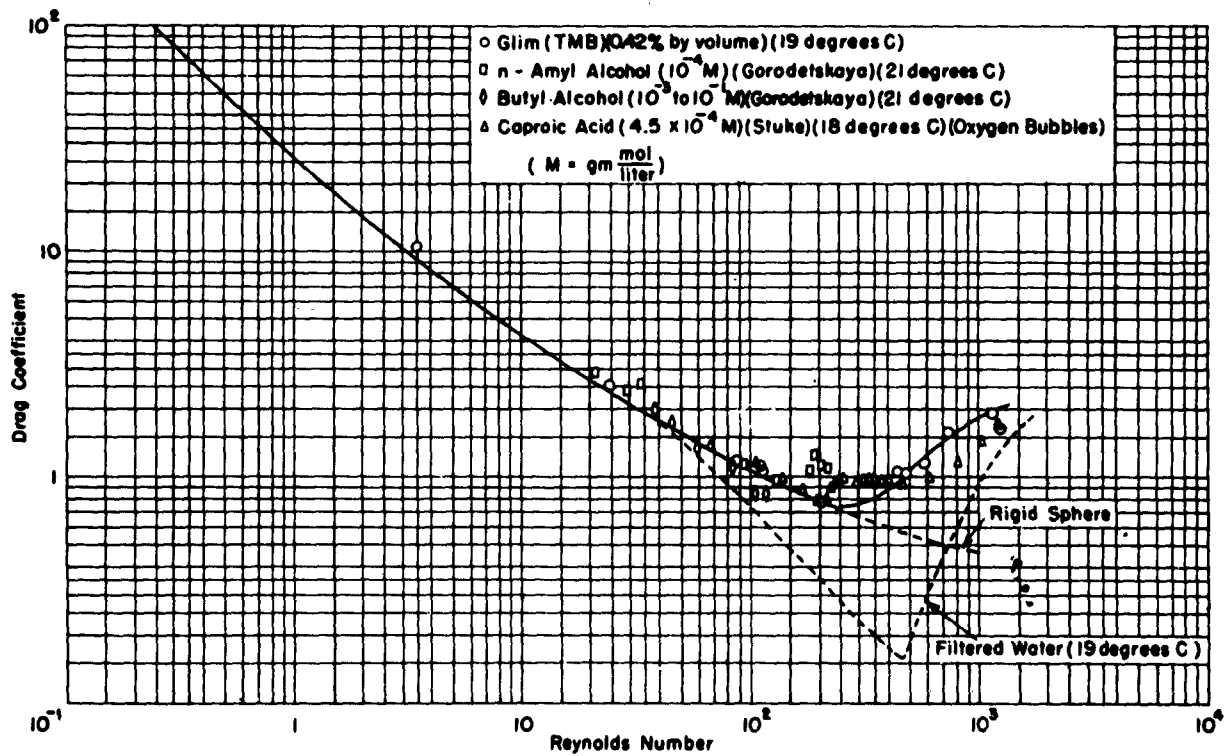


Figure 29 - Drag Coefficient as a Function of Reynolds Number for Bubbles Rising at Their Terminal Velocity in Water Containing Various Surface-Active Materials

range of bubble sizes did not extend fully into the region of spherical cap bubbles, it is quite certain that the presence of surface-active substances will not alter their rate of rise, which was shown to be independent of all physical properties of the liquid.

The difference in behavior of the air bubbles must be sought in the behavior at the surface. A high concentration of molecules of the surface-active substances will be found at the surface of the bubble. As for the case of the tap water, these molecules would travel with the bubble and would impart, in effect, a "rigid" surface to the bubble, that is to say, would impose the condition of zero velocity at the boundary.

Thus, the results of the tests given in Figure 29 show that these surface-active substances increase the drag of the bubbles (in the region of bubbles having equivalent radii of 0.08 to 0.30 cm); beyond the critical concentrations, any increase in concentration has relatively little influence on the drag of the bubbles.

BUBBLES AS RIGID BODIES

In previous sections it was shown that bubbles rising in pure liquids* behave essentially like fluid bodies over a large range of bubble size, but that below a certain critical size (the size being different for various liquids), the bubbles behave like rigid bodies, that is to say, the drag of the bubbles equals that of corresponding rigid bodies.

A possible explanation of the anomaly of behavior of the gas bubbles is given by Boussinesq's dynamic surface tension.⁵ As pointed out under "Theoretical Solutions," for small bubbles the effect of the dynamic increment of surface tension increases the drag to the value of corresponding rigid bodies. With increase in bubble size, this effect becomes negligible and the drag of the bubble equals that of a fluid body. The dynamic increment includes a constant of proportionality (surface viscosity) which is a function of the two fluids composing the interface. Therefore, for air bubbles, the transition region from "rigid" to fluid bodies would be different for the various liquids. There is, however, no experimental evidence that dynamic surface tension, as postulated by Boussinesq, exists.

From a "hydrodynamic" point of view, the reason for the transition of the bubbles to "rigid" bodies is not clear. As mentioned earlier, the mere inclusion of surface tension as a pressure drop in the boundary conditions does not alter the analytical solution for fluid spheres. Hence, it appears that the presence of surface tension should have no effect on the motion of the bubble, except in maintaining the spherical shape. Thus the anomalous behavior of the bubbles cannot be explained in terms of "hydrodynamics," but must be sought in terms of a surface phenomenon. If it could be shown analytically (if only in the region of slow flow) that equality of drag of corresponding rigid bodies and bubbles also implies equality of boundary conditions at the surface, then, as in the case of rigid bodies, the velocity of the entire

*Mixtures such as the 13 percent ethyl alcohol-water mixture are included in this category.

surface of the bubble must vanish. The surface must then be able to hold molecules of the pure liquid, just as in the case of tap water and surface-active substances the surface attracts and holds a high concentration of particles or molecules of the surface-active substances. The molecules at the surface would travel with the bubble and hence would, in effect, give the same boundary conditions as a rigid surface. As the shear forces become larger in comparison to the forces holding the molecules at the surface, "rigidity" at the surface cannot be maintained; circulation inside the bubble ensues and the drag of the bubble becomes smaller as compared to that of a rigid body.

In addition, since it was not possible to correlate the results of the experiments on the motion of bubbles in the gravity pressure field in terms of nondimensional parameters formed from the usual liquid properties (viscosity, surface tension, density), further work on freely rising bubbles is necessary before the results obtained from such tests can be utilized and before the more complicated behavior of bubbles in variable pressure gradients can be understood. Particularly, an understanding of the reason for the transition of bubbles from fluid to "rigid" bodies as well as a criterion for this transition is most desirable, since such transition might be influenced by the magnitude of the pressure gradient. In the region of Reynolds numbers where the bubbles behave like rigid spheres, the pressure gradient probably has no effect on the drag coefficient.

SUMMARY

As the size of the bubbles was increased in the tests, a change in bubble shape from spherical to ellipsoidal to spherical cap shape was observed in all liquids. The volumes at which these transitions occur, however, varied with the properties of the liquid. For spherical bubbles of given volume the results show that the viscosity of the liquid is the most important property determining the rate of rise. For ellipsoidal bubbles, the surface tension assumes greater importance. Spherical cap bubbles rise independently of the properties of the liquid.

The results show that the motion of air bubbles rising at their terminal velocity in a gravity field cannot be described completely by use of dimensionless parameters formed from the usual liquid properties (viscosity, surface tension, density), the equivalent radius of the bubble, the acceleration of gravity, and the terminal velocity.

The drag coefficients of tiny spherical bubbles coincide with those of corresponding rigid spheres. With increase in bubble size, a decrease in the drag as compared to that of rigid spheres occurs in some liquids. This change in the drag is due to the development of circulation inside the bubble. The drag curves of the spherical bubbles rising in various liquids fall between two limiting curves, namely the drag curves of rigid and fluid spheres, respectively. It was not possible to determine a criterion for the transition region of the bubbles from "rigid" to fluid spheres.

The region of ellipsoidal bubbles extends over different ranges of Reynolds numbers

for the various liquids. For liquids of low "M" number (say less than 10^{-3}), a minimum in the drag curve is reached at Reynolds numbers of the order of 250. These minima occur near the transition from spherical to ellipsoidal shape. Such minima in the drag curve are not obtained for liquids of high "M" number. For the liquids used, transition to spherical caps is completed at a Weber number of about 20.

The drag coefficients of spherical cap bubbles are independent of bubble size and have a constant value of 2.6. The rate of rise of these bubbles as a function of the equivalent radius is given by the experimentally determined relation:

$$U = 1.02 \sqrt{gr_c}$$

For bubbles (ranging in equivalent radius from 0.03 to 0.25 cm) rising in tap water, an increased drag as compared to bubbles in clean (filtered or distilled) water was observed. The presence of certain surface-active substances in the water similarly increases the drag of bubbles (ranging in equivalent radius from 0.03 to 0.30 cm) as compared to bubbles in pure water. Beyond a certain critical concentration of these surface-active substances, an increase in concentration has relatively little influence on the drag of the bubbles.

Tests to determine the effect of the container walls on the velocity of rise indicate the absence of such effect for the range of bubble volumes and container sizes tested.

ACKNOWLEDGMENTS

The authors wish to acknowledge the suggestions of Dr. Lawrence M. Kushner, National Bureau of Standards, regarding the explanation of the behavior of bubbles in tap water and in water containing surface-active materials.

REFERENCES

1. Rosenberg, Benjamin, "The Drag and Shape of Air Bubbles Moving in Liquids," TMB Report 727, September 1950.
2. Lamb, Horace, "Hydrodynamics," Dover Publications, New York, 1945, p. 599.
3. Hadamard, J. "Mouvement permanent lent d'une sphère liquide et visqueuse dans un liquide visqueux," Comptes Rendus, Acad. Sci., Paris, 1911, Vol. 152, pp. 1735-1738.
4. Rybczynski, W., "Über die fortschreitende Bewegung einer flüssigen Kugel in einem zähen Medium," Bulletin Academie de Sciences de Cracovie (Series A), 1911, pp. 40-46.
5. Boussinesq, J., "Vitesse de la chute lente, devenue uniforme, d'une goutte liquide sphérique, dans un fluide visqueux de poids spécifique moindre," Ann. de Chimie et de Phys., 1913, Vol. 29, pp. 364-372, or Comptes Rendus, 1913, Vol. 156, pp. 1124-1129, also see Comptes Rendus, 1913, Vol. 157, pp. 313-318.

6. Bulletin of the National Research Council, No. 84, February 1932, pp. 297-308.
7. Theremin, "Recherches sur la figure et le mouvement d'une bulle d'air dans un liquide de densité constante," Crelle Journal, 1830, Vol. 5, pp. 93-101, 374-379.
8. Schmidt, Ernst, "Ähnlichkeitstheorie der Bewegung der Flüssigkeitsgasgemischen," 1934, VDI Forschungsheft 365, or "Über die Bewegungsverhältnisse von Gas-Flüssigkeitsgemischen," 1933, VDI, 71st Hauptversammlung.
9. Allen, H.S., "The Motion of a Sphere in a Viscous Fluid," Phil. Mag., 1900, Vol. 50, pp. 323-338, 519-534.
10. Arnold, H.D., "Limitations Imposed by Slip and Inertia Terms upon Stokes's Law for the Motion of Spheres through Liquids," Phil. Mag., 1911, Vol. 22, pp. 755-775.
11. Bond, W.N., and Newton, Dorothy A., "Bubbles, Drops, and Stokes' Law," Phil. Mag., 1928, Vol. 5, pp. 794-800.
12. Miyagi, Otagoro, "The Motion of an Air Bubble Rising in Water," Tech. Reports of the Tohoku Imperial University, 1925, Vol. 5, pp. 135-167.
13. Exner, Felix M., "Über die Aufstiegeschwindigkeit von Luftblasen im Wasser," Phys. Zeitschrift, 1927, Vol. 28, pp. 825-829.
14. Hoefler, Kurt, "Untersuchungen über die Strömungsvorgänge im Steigrohr eines Druckluft-Wasserhebers," VDI Mitteilungen über Forschungsarbeiten, 1913, No. 138, pp. 1-12.
15. Owens, J.S., "Experiments on Air Lift Pumping," Engineering (Great Britain), September 1921, Vol. 112, pp. 458-461.
16. Ward, C.N., and Kessler, L.H., "Experimental Study of Air Lift Pumps and Application of Results to Design," Bulletin of University of Wisconsin, Eng. Series, 1924, Vol. 9, No. 4, Ser. No. 1265,
17. O'Brien, Morrrough P., and Gosline, James E., "Velocity of Large Bubbles in Vertical Tubes," Ind. and Eng. Chem, December 1935, Vol. 27, No. 12, pp. 1436-1440.
18. Luchsinger, W., "Disperse Gases III: Bubble Size and Time of Rise," (Kolloid-Zeitschrift, 1937, Vol. 80, pp. 180-182), TMB Translation 115, September 1943.
19. Göpfert, Herbert, "The Velocity of Ascension of Gas Bubbles in Water," Great Britain, Department of Scientific Research and Experiment. Translation No. 341, April 1946.
20. Napier, D.H., "Ascent of Air Bubbles in Air-Saturated Water," Great Britain Imperial College, Department of Chemical Engineering, Gas Absorption Working Party Report No. 10, March 1949.
21. Carstensen, E.L., and Foldy, L.L., "Propagation of Sound through a Liquid Containing Bubbles," Part II, OSRD No. 3872, Section No. 6.1-sr 1130-1629, June 1944.

22. Davies, R.M. and Taylor Geoffrey, "The Mechanics of Large Bubbles Rising Through Extended Liquids and Through Liquids in Tubes," Proc. Royal Soc., London, 1950, Vol. 200, Series A, pp. 375-390.

23. Temperley, H.N.V. and Chambers, L.I.G., "The Rate of Rise of Large Volumes of Gas in Water," Great Britain, Admiralty Underworks Report No. AUW/TRI 63/RF20, Under 125, January 1945.

24. Bryn, T., "Speed of Rise of Air Bubbles in Liquids," (Forschung, January-February 1938, Vol. 4, No. 1), TMB Translation 132, June 1949.

25. Robinson, J.V., "The Rise of Air Bubbles in Lubricating Oils," J. Phys. and Colloid Chem., March 1947, Vol. 51, No. 2, pp. 431-437, or NACA Tech. Note 2033, February 1950.

26. Pickert, F., "Das Aufsteigen von Gasblasen in Flüssigkeiten" (The Rise of Gas Bubbles in Liquids), Forschung (B), November-December 1932, Vol. 3, pp. 308-309.

27. Pekeris, C.L., "The Rate of Rise and Diffusion of Air Bubbles in Water," OSRD No. 976, Section No. C4-sr 20-326, October 1942.

28. Worster, R.C., "On the Rising Velocity of Air Bubbles in Water," British Hydro-mechanics Research Association, Tech. Note 1, July 1948.

29. Datta, R.L., Napier, D.H., and Newitt, D.M., "The Properties and Behaviour of Gas Bubbles Formed at a Circular Orifice," Trans. Institution of Chemical Engineers (Great Britain), 1950, Vol. 28, pp. 14-26.

30. Gorodetskaya, A., "The Rate of Rise of Bubbles in Water and Aqueous Solutions at Great Reynolds Numbers," J. Phys. Chem. (U.S.S.R.), 1949, Vol. 23, pp. 71-77, (in Russian).

31. Birkhoff, G. and Caywood, T.E., "Fluid Flow Patterns," BuOrd Contract NOrd-8555, Task B, NavOrd Report 447, 1 February 1947. Also J. App. Phys., July 1949, Vol. 20, No. 7.

32. Prandtl, Ludwig, "Führer durch die Strömungslehre," F. Vieweg and Sohn, Brunswick (Germany), 1942, p. 293.

33. Shabalin, K.N., Krylov, S.F., and Oborin, V.I., "The Rate of Absorption under Conditions of Bubbling," J. of Chemical Industry USSR, 1939, Vol. 16, p. 10 (in Russian).

34. Coppock, P.D. and Meiklejohn, G.T., "The Behaviour of Gas Bubbles in Relation to Mass Transfer," Trans. Institution of Chemical Engineers (Great Britain), 1951, Vol. 29, No. 1, pp. 75-86.

35. Franz, G.F., "Shape and Path of Entrained Air Bubbles around Elliptic Cylinder in Water," M.S. Thesis, University of Colorado, 1951. (Copy available in TMB Library); see also TMB Report C-482, May 1952.

36. Miyagi, O., "Measurement of Stream Velocity by Air Bubbles," J. Soc. Mech. Engrs., (Japan), July 1929, Vol. 32, No. 147, pp. 305-313.

37. Zdonik, S.B., "The Liquid-Film Coefficient and the Mechanism of Bubble Absorption," M.S. Thesis, Department of Chem. Eng., Massachusetts Institute of Technology, 1942.
38. Redding, E.M., "Absorption of Small Gas Bubbles in Salt Water," D.Sc. Thesis, Department of Chem. Eng., Massachusetts Institute of Technology, 1942.
39. Oyama, Y. and Iwase, K., "Studies on Absorption of Gas in Form of Bubbles," Scientific Papers, Inst. Phys. and Chem. Research (Tokyo), January 1939, Vol. 35, pp. 131-157.
40. Stuke, Bernward, "Das Verhalten der Oberfläche von sich in Flüssigkeiten bewegenden Gasblasen, Die Naturwissenschaften, 1952, Vol. 39, No. 14, pp. 325-326.
41. Dubs, R., "Ueber die Bewegung von Luftblasen in ruhenden und fliessendem Wasser," Schweizerische Bauzeitung, 1931, Vol. 97, No. 14, pp. 169-173 and No. 15, pp. 183-187.
42. Ladenburg, Rudolf, "Über den Einfluss von Wänden auf die Bewegung einer Kugel in einer reibenden Flüssigkeit," Ann. der Phys. 4th Series, 1907, Vol. 23, pp. 447-458.
43. McNown, J.S., et al, "Influence of Boundary Proximity on the Drag of Spheres," State University of Iowa Reprints in Engineering, No. 81, 1948.
44. Möller, Wilhelm, "Experimentelle Untersuchungen zur Hydrodynamik der Kugel," Phy. Zeitschrift, 1938, Vol. 39, No. 2, pp. 57-80.
45. Gibson, A.H., "On the Motion of Long Air Bubbles in a Vertical Tube," Phil. Mag., 1913, Vol. 26, No. 156, pp. 952-965.
46. Hattori, Sin-iti, "On the Motion of a Cylindrical Bubble in a Tube and Its Application to the Measurement of the Surface Tension of a Liquid," Tokyo Imperial University, Aeronautical Research Institute, 1935, No. 115, pp. 163-195.
47. Dumitrescu, D.T., "Strömung an einer Luftblase im senkrechten Rohr," ZAMM, 1943, Vol. 23, No. 3, pp. 139-149.
48. Goldstein, Sidney, "Modern Developments in Fluid Dynamics," Oxford University Press, London, 1950, Vol. I, p. 16.
49. Garner, F.H., "Diffusion Mechanism in the Mixing of Fluids," Trans. Institution of Chemical Engineers (Great Britain), 1950, Vol. 28, pp. 88-96.
50. Schiller, Ludwig, "Handbuch der Experimentalphysik," Leipzig (Germany), 1932, Vol. IV, Part 2, p. 386.

APPENDIX

RATE OF RISE OF GAS BUBBLES IN DISTILLED WATER

It is of interest to compare the rate of rise of bubbles of different gases in distilled water. As pointed out under "Previous Experimental Work," a number of tests have been conducted using oxygen bubbles. In Figure 80, the velocity of air and of oxygen bubbles in distilled water is given as a function of the equivalent radius of the bubble. The results indicate no significant difference in the rate of rise of the bubbles composed of oxygen and those of air.

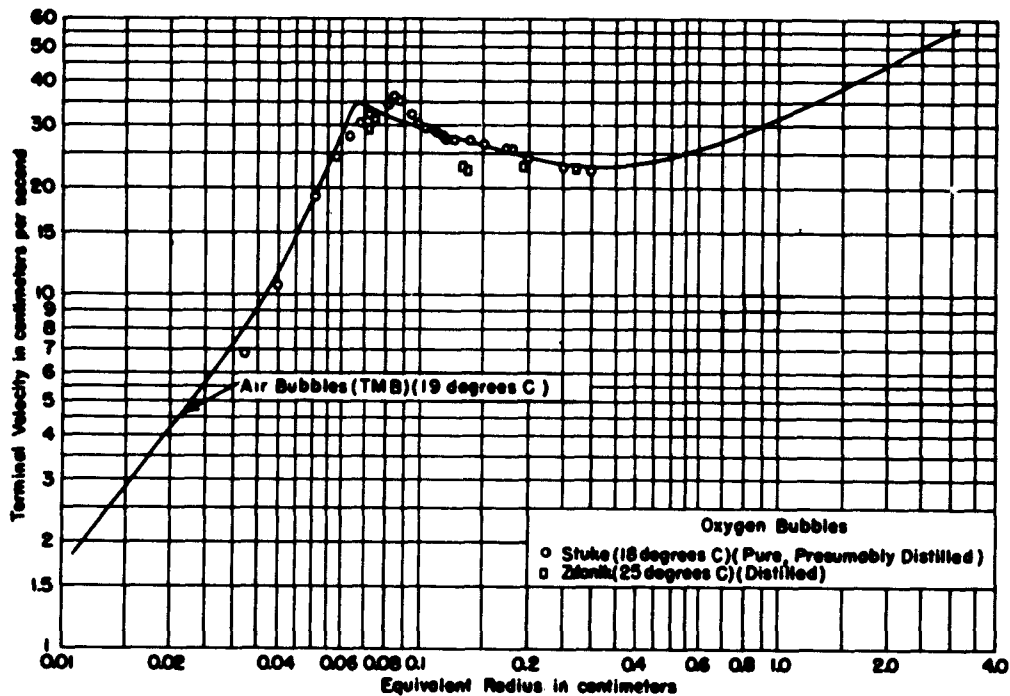


Figure 80 - Terminal Velocity of Gas Bubbles in Distilled Water as a Function of Bubble Size

INITIAL DISTRIBUTION

Copies

- 20 Chief, Bureau of Ships, Technical Library (Code 327), for distribution:
 - 5 Technical Library
 - 1 Civilian Consultant to Chief (Code 106)
 - 2 Research and Development (Code 300)
 - 1 Applied Science (Code 370)
 - 1 Noise, Shock, and Vibration Branch (Code 371)
 - 1 Ship Design (Code 410)
 - 2 Preliminary Design and Ship Protection (Code 420)
 - 1 Preliminary Design (Code 421)
 - 1 Model Basin Liaison (Code 422)
 - 1 Submarines (Code 515)
 - 1 Minesweeping (Code 520), for Mr. B. Rosenberg
 - 1 Propellers and Shafting (Code 554)
 - 1 Sonar (Code 845)
 - 1 Sonar Support (Code 847)
- 4 Chief, Bureau of Ordnance, Underwater Ordnance
 - 2 Code Re6
 - 2 Code Re3
- 3 Chief, Bureau of Aeronautics
 - 2 Aero and Hydrodynamics Branch (DE-8)
 - 1 Applied Mathematics Branch (RS-7)
- 5 Chief of Naval Research
 - 3 Fluid Mechanics Branch (Code N426)
 - 1 Naval Sciences Division (Code 460)
 - 1 Undersea Warfare Division (Code 466)
- 1 Director, Office of Naval Research, Branch Office, 346 Broadway, New York 13, N.Y.
- 1 Director, Office of Naval Research, Branch Office, 1030 E. Green Street, Pasadena 1, Calif.
- 1 Director, Office of Naval Research, Branch Office, 1000 Geary Street, San Francisco 9, Calif.
- 1 Director, Office of Naval Research, Branch Office, The John Crerar Library Building, Tenth Floor, 86 E. Randolph Street, Chicago 1, Ill.
- 1 Director, Office of Naval Research, Branch Office, 150 Causeway Street, Boston 14, Mass.
- 1 Officer in Charge, Office of Naval Research, Branch Office, London, England, Navy 100, Fleet Post Office, New York, N.Y.
- 1 Commander, Portsmouth Naval Shipyard, Portsmouth, N.H.
- 1 Commander, Puget Sound Naval Shipyard, Bremerton, Wash.

Copies

- 1 Commander, Philadelphia Naval Shipyard, Naval Base, Philadelphia 12, Pa.
Attn: Service Library
- 1 Commander, Boston Naval Shipyard, Code 263a, Boston 29, Mass.
- 2 Commander, U.S. Naval Ordnance Laboratory, White Oak, Silver Spring 19, Md.
- 1 Commander, U.S. Naval Ordnance Test Station, Pasadena Annex, 3202 E.
Foot Hill Blvd., Pasadena, Calif.
- 1 Commander, U.S. Naval Ordnance Test Station, China Lake, Inyokern, Calif.
- 1 Commanding Officer, U.S. Naval Medical Research Institute, National Naval
Medical Center, Bethesda 14, Md.
- 1 Commanding Officer and Director, U.S. Navy Electronics Laboratory, San Diego,
Calif.
- 1 Commander, U.S. Naval Air Development Center, Johnsville, Pa. Attn: Library
- 1 Director, U.S. Naval Research Laboratory, Anacostia, Washington 25, D.C.
- 1 Chairman, Research and Development Board, Department of Defense Building,
Washington 25, D.C.
- 1 The Director, Waterways Experiment Station, Corps of Engineers, U.S. Army,
Vicksburg, Miss.
- 1 Director, National Bureau of Standards, Washington 25, D.C.
- 1 Director, Aeronautical Research, National Advisory Committee for Aeronautics,
1724 F Street, N.W., Washington 25, D.C.
- 1 Director, Oak Ridge National Laboratory, P.O. Box P, Oak Ridge, Tenn.
- 1 Director, Applied Physics Division, Sandia Laboratory, Albuquerque, N.M.
- 1 Chief, Hydraulic Data Branch, Tennessee Valley Authority, Knoxville, Tenn.
- 1 Director of Military Application, U.S. Atomic Energy Commission, 1901 Consti-
tution Avenue, Washington 25, D.C. Attn: Mr. Paul C. Fine, Tech. Asst.
- 1 Head, Technical Reference Section, U.S. Department of the Interior, Bureau of
Reclamation, Denver Federal Center, Denver, Colo.
- 1 Editor, Bibliography of Technical Reports, Office of Technical Services, U.S.
Department of Commerce, Washington 25, D.C.
- 1 Editor, Technical Data Digest, Armed Services Technical Information Agency,
Document Service Center, U.B. Building, Dayton 2, Ohio
- 1 Editor, Applied Mechanics Reviews, Midwest Research Institute, 4049 Pennsyl-
vania, Kansas City 2, Mo.
- 1 Editor, Aeronautical Engineering Reviews, 2 E. 64th Street, New York 21, N.Y.
- 1 Editor, Engineering Index, 29 W. 39th Street, New York 18, N.Y..

Copies

- 1 Librarian, American Society of Mechanical Engineers, 20 W. 39th Street,
New York 18, N.Y.
- 1 Librarian, American Society of Civil Engineers, 33 W. 39th Street, New York 18,
N.Y.
- 1 Librarian, American Chemical Society, 1155 16th Street, N.W., Washington 6,
D.C.
- 1 Librarian, Franklin Institute, Parkway at 20th Street, Philadelphia, Pa.
- 1 Librarian, Institute of the Aeronautical Sciences, 2 E. 63rd Street, New York
21, N.Y.
- 1 Librarian, Mechanics Research Library, Illinois Institute of Technology,
Tech. Center, Chicago 16, Ill.
- 1 Librarian, The John Crerar Library, 86 E. Randolph Street, Chicago 1, Ill.
- 1 Librarian, Pacific Aeronautical Library, 7660 Beverly Blvd., Los Angeles 36,
Calif.
- 1 Librarian, American Institute of Physics, 57 E. 55th Street, New York 22, N.Y.
- 1 Librarian, Cornell University, College of Engineering, Ithaca, N.Y.
- 1 Librarian, Rensselaer Polytechnic Institute, Troy, N.Y.
- 1 Librarian, Foster-Wheeler Corporation, 165 Broadway, New York 6, N.Y.
- 1 Librarian, Worthington Corporation, Harrison, N.J.
- 1 Librarian, American Machine and Foundry Company, 170 53rd Street, Brooklyn
32, N.Y.
- 1 Librarian, Shell Development Company, Emeryville, Calif.
- 1 Librarian, University of Illinois, Urbana, Ill.
- 1 Librarian, Babcock and Wilcox Company, Research and Development Department,
Alliance, Ohio
- 1 Main Library, Carnegie Institute of Technology, Pittsburgh, Pa.
- 1 Librarian, Institution of Chemical Engineers, 56 Victoria Street, Westminster,
London SW1, England
- 1 Librarian, The Institute of Physics, 47 Belgrave Square, London SW1, England
- 2 Editor, Physics Abstracts, Institute of Electrical Engineers, Savoy Place,
London W.C. 2, England
- 1 Editor, Bulletin of the British Hydromechanics Research Association, Nettes-
well Road, Harlow, Essex, England
- 1 Editeur, La Houille Blanche, Boîte Postale 41, Grenoble, France

Copies

- 1 Director, Armour Research Foundation, 35 W. 33rd Street, Chicago 18, Ill.
- 1 Director, Alden Hydraulic Laboratory, Worcester Polytechnic Institute, Worcester 2, Mass.
- 2 Director, Applied Physics Laboratory, Johns Hopkins University, 8621 Georgia Avenue, Silver Spring, Md.
- 1 Director, Fluid Mechanics Laboratory, Columbia University, New York 27, N.Y.
- 1 Director, Fluid Mechanics Laboratory, University of California, Berkeley 4, Calif.
- 1 Director, Hydraulic Laboratory, Carnegie Institute of Technology, Pittsburgh 13, Pa.
- 1 Director, Hydraulic Laboratory, University of Colorado, Boulder, Colo.
- 1 Director, Hydraulic Research Laboratory, University of Connecticut, Box U-37, Storrs, Conn.
- 1 Hydrodynamics Laboratory, Attn: Executive Committee, California Institute of Technology, 1201 E. California Street, Pasadena 4, Calif.
- 1 Director, Scripps Institute of Oceanography, University of California, La Jolla, Calif.
- 1 Harvard University, School of Engineering, Cambridge 38, Mass.
- 1 Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.
- 1 Head, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Director, Fluid Mechanics Laboratory, New York University, New York 53, N.Y.
- 1 Supervisor of Shipbuilding, USN, and Naval Inspector of Ordnance, New York Shipbuilding Corporation, Camden, N.J.
- 1 Director, Robinson Hydraulic Laboratory, Ohio State University, Columbus, Ohio
- 1 Director, Hydraulics Laboratory, Pennsylvania State College, State College, Pa.
- 1 Department of Mechanical Engineering, Stanford University, Calif.
- 1 Director, Experimental Towing Tank, Stevens Institute of Technology, 711 Hudson Street, Hoboken, N.J.
- 1 Dean, School of Engineering, University of Texas, Austin, Texas
- 1 Director of Research, Vickers Incorporated, Detroit, Mich.
- 1 Director, Woods Hole Oceanographic Institution, Woods Hole, Mass. Attn: Mr. Allyn C. Vine, Mr. R.L. Rather, Mr. Von Arx
- 1 Director, Hydraulic Laboratory, University of Wisconsin, Madison 6, Wis.
- 1 Director, Hydraulics Laboratory, University of Washington, Seattle 5, Wash.

Copies

- 1 **Development Contract Administrator, Ordnance Research Laboratory, for
Director, Ordnance Research Laboratory, Pennsylvania State College,
State College, Pa.**
- 1 **Head, Department of Chemical Engineering, Massachusetts Institute of Technolo-
gy, Cambridge 39, Mass.**
- 1 **Virginia Polytechnic Institute, Department of Applied Mechanics, Blacksburg, Va.**
- 1 **Prof. G. Birkhoff, Department of Mathematics, Harvard University, Cambridge 38,
Mass.**
- 1 **Prof. R.C. Binder, Department of Mechanical Engineering, Purdue University,
Lafayette, Ind.**
- 1 **Prof. K.J. DeJuhasz, Pennsylvania State College, State College, Pa.**
- 1 **Prof. S.A. Guerrieri, Division of Chemical Engineering, University of Delaware,
Newark, Delaware**
- 1 **Prof. W.S. Hamilton, Technical Institute, Northwestern University, Evanston, Ill.**
- 1 **Prof. A.D. Hay, School of Engineering, Princeton University, Princeton, N.J.**
- 1 **Dr. A.T. Ippen, Director, Hydrodynamics Laboratory, Department of Civil and
Sanitary Engineering, Massachusetts Institute of Technology, Cambridge 39,
Mass.**
- 1 **Inspector of Naval Material, 1206 S. Santee Street, Los Angeles, Calif. for
Dr. R.T. Knapp, Hydrodynamics Laboratory, California Institute of Tech-
nology, Pasadena 4, Calif.**
- 1 **Mr. C.A. Lee, Research and Development Laboratories, Kimberly-Clark Corpora-
tion, Neenah, Wis.**
- 1 **Dr. J.H. McMillen, National Science Foundation, Washington, D.C.**
- 1 **Dr. A. May, Aerodynamics Division, U.S. Naval Ordnance Laboratory, White Oak,
Silver Spring 19, Md.**
- 1 **Dr. M.S. Plesset, Hydrodynamics Laboratories, California Institute of Technology,
Pasadena 4, Calif.**
- 1 **Dr. H. Rouse, Director, Iowa Institute of Hydraulic Research, State University
of Iowa, Iowa City, Iowa**
- 1 **Dr. J.V. Wehausen, Executive Editor, Mathematical Reviews, 80 Waterman Street,
Providence, R.I.**
- 1 **Dr. F.E. Seely, Fluid Mechanics and Hydraulics Laboratory, University of Illinois,
Urbana, Ill.**
- 1 **Prof. J.K. Vennard, Director, Hydraulic Laboratory, Stanford University, Calif.**
- 1 **Prof. C.A. Shreeve, Jr. Mechanical Engineering Department, University of Mary-
land, College Park, Md.**

Copies

- 1 Dr. E.M. Redding, care Charles F. Kettering Foundation, Box 48, Far Hills
Branch Post Office, Dayton 9, Ohio
- 1 Dean M.P. O'Brien, Department of Engineering, University of California,
Berkeley 4, Calif.
- 1 Prof. J.H. Rushton, Department of Chemical Engineering, Illinois Institute of
Technology, Chicago 16, Ill.
- 1 Dr. A. Nadai, 136 Cherry Valley Road, Pittsburgh 21, Pa.
- 1 Prof. F.K. Teichmann, Department of Aeronautical Engineering, New York
University, University Heights 53, N.Y.
- 1 Prof. A.L. Jorissen, Head, Department of Hydraulics and Hydraulic Engineering,
Cornell University, Ithaca, N.Y.
- 1 Prof. F.N. Peebles, Department of Chemical Engineering, University of
Tennessee, Knoxville 16, Tenn.
- 1 Mr. D.B. Bogart, U.S. Geological Survey, Hydrological Unit, Post Office Build-
ing, Albany, N.Y.
- 1 Dr. W.J. Ford, Department of Aeronautical Engineering, Cornell University,
Ithaca, N.Y.
- 1 Dr. R.R. Hughes, Shell Development Company, Emeryville, Calif.
- 1 Directeur, Laboratoire Dauphinois d'Hydraulique des Ateliers Neyrpic, Avenue
de Beauvert, Grenoble (Isère), France
- 1 The Director and Secretary, The Science Museum, South Kensington, London
S.W.7, England
- 1 Director, British Shipbuilding Research Association, 5 Chesterfield Gardens,
Curzon Street, London W.1, England
- 1 Director, Christian Michelson Institute, Bergen, Norway
- 1 Head, Aerodynamics Division, National Physical Laboratory, Teddington,
Middlesex, England
- 1 Office of Scientific Attaché of Netherlands Embassy, 1470 Euclid Street, N.W.,
Washington, D.C.
- 1 Canadian National Research Establishment, Halifax, Nova Scotia, Canada
- 1 Admiralty Research Laboratory, Teddington, Middlesex, England
- 1 Armament Research Establishment, Fort Halstead, Hants, England
- 1 Prof. J. Ackeret, Institut für Aerodynamik der Eidgenössischen Technischen
Hochschule, Zurich, Switzerland
- 1 Dr. S. Goldstein, Haifa Institute of Technology, Haifa, Israel
- 1 Dr. J. Okaba, The Research Institute for Applied Mechanics, Kyushu Univer-
sity, Hakozaki-machi, Fukuoka-shi, Japan

Copies

- 1 Dr. F.C. Roesler, Physikalisches Institut, Tech. Hochschule Graz, Rechbauer-
strasse 12, Graz, Austria
- 1 Mr. B. Stuke, Physikalisches-Chemisches Institut der Universität München, Munich,
Germany
- 1 Mr. P.D. Coppock, Research and Development Department, The Distillers'
Company Ltd., Great Burgh, Epsom, Surrey, England
- 1 Mr. H. Verschoor, N.V. De Dataafsche Petroleum Maatschappij, The Hague,
Netherlands
- 1 Prof. F. Numachi, Director of the Institute of High-Speed Mechanics, Tohoku
University, Sendai, Japan
- 1 Mr. G. Halbronn, Etablissement NEYRPIC, Grenoble, France
- 1 Director, Hydrodynamics Laboratory, National Research Council, Ottawa, Canada
- 1 Australian Scientific Liaison Office, 1800 K Street, N.W., Washington 25, D.C.
- 9 British Joint Services Mission (Navy Staff), P.O. Box 165, Benjamin Franklin
Station, Washington, D.C.
- 1 Dr. L.G. Straub, Director, St. Anthony Falls Hydraulic Laboratory, University
of Minnesota, Minneapolis 14, Minn.
- 1 Dr. V.L. Streeter, Director, Fundamental Fluid Research, Illinois Institute of
Technology, Technology Center, Chicago 16, Ill.